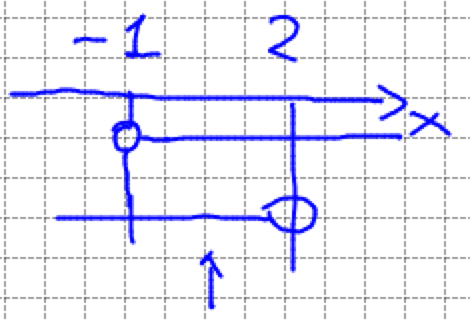
ESEMPIO

$$\log_{\frac{1}{3}}(x+1) - 2\log_{\frac{1}{3}}(2-x) \leq 1$$

C.E. $\left\{ x \in \mathbb{R} \;\middle|\; \begin{cases} x+1>0 \\ 2-x>0 \end{cases} \right. \Rightarrow \begin{cases} x>-1 \\ x<2 \end{cases}$



C.E. $(-1;2)$

$$\begin{cases} \log_{\frac{1}{3}}(x+1) - 2\log_{\frac{1}{3}}(2-x) \leq 1 \\ -1<x<2 \end{cases}$$

$$\log_{\frac{1}{3}}(x+1) - \log_{\frac{1}{3}}(2-x)^2 \leq 1$$

$$\begin{cases} \log_{\frac{1}{3}}\dfrac{(x+1)}{(2-x)^2} \leq 1 \\ -1<x<2 \end{cases} \qquad \begin{cases} \dfrac{x+1}{(2-x)^2} \geq \dfrac{1}{3} \\ -1<x<2 \end{cases}$$

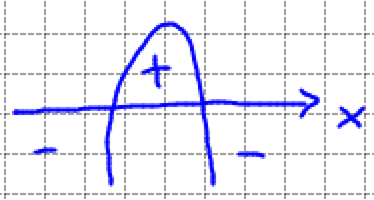
$$\begin{cases} \dfrac{3x+3-4-x^2+4x}{3(2-x)^2} \geq 0 \\ -1<x<2 \end{cases}$$

$$\begin{cases} \dfrac{-x^2+7x-1}{3(2-x)^2} \geq 0 \\ -1<x<2 \end{cases}$$

N $\quad -x^2+7x-1 \geq 0 \qquad -x^2+7x-1=0$

$$x_{1,2} = \frac{-7 \pm \sqrt{49-4}}{-2} = \frac{-7 \pm \sqrt{45}}{-2}$$

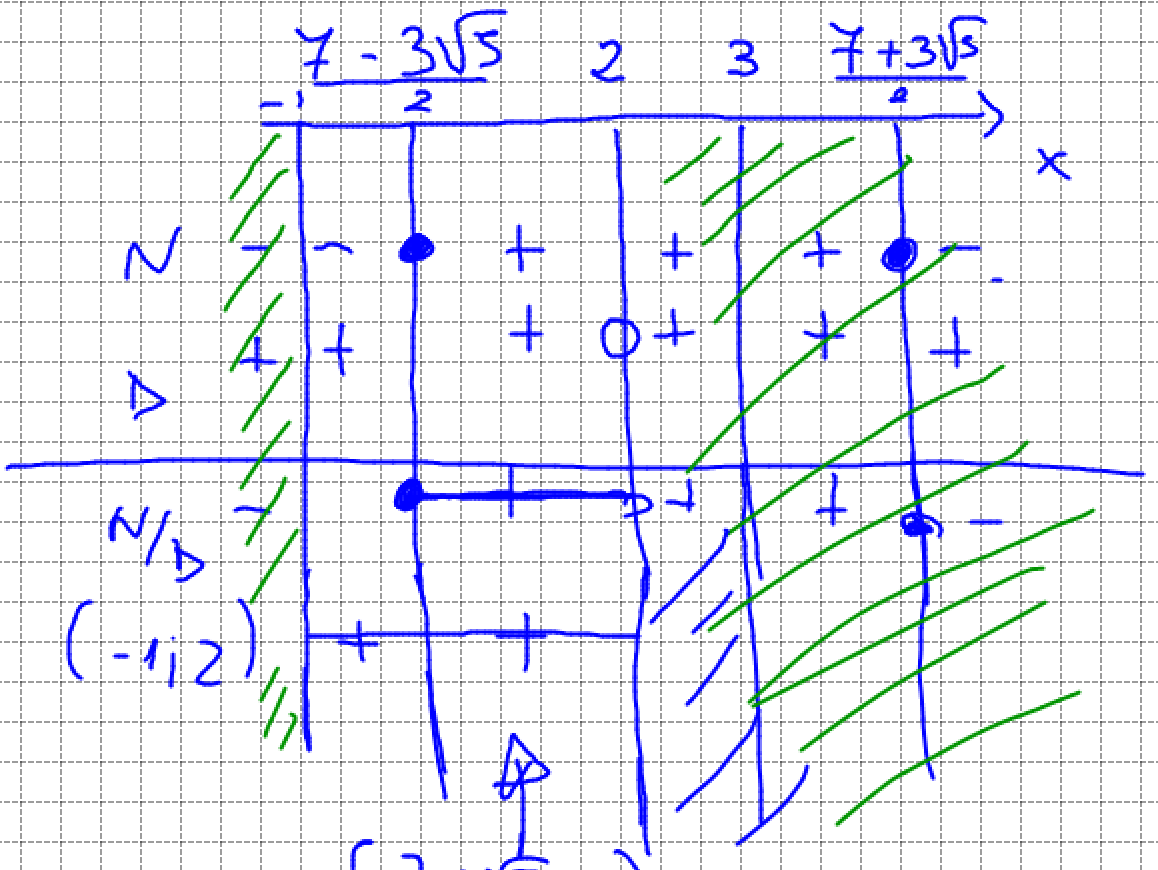
$$x_{1,2} = \begin{cases} \dfrac{7+3\sqrt{5}}{2} \\ \dfrac{7-3\sqrt{5}}{2} \end{cases}$$



D $\quad 3(2-x)^2 > 0$

$3>0$ sempre

$(2-x)^2>0$ sempre + escluso $x=2$



$$\left[\frac{7-3\sqrt{5}}{2}; 2\right)$$

ES 644 PAG 613

$$3\log_5(x-4) > \frac{6}{\log_5(x-4)+1}$$

$$C.E. = \left\{ x \in \mathbb{R} \Big/ \begin{cases} x-4>0 \\ \log_5(x-4)+1 \neq 0 \end{cases} \right\} =$$

$$\begin{cases} x>4 \\ \log_5(x-4) \neq -1 \end{cases} \Leftrightarrow \begin{cases} x>4 \\ x-4 \neq \frac{1}{5} \end{cases} \Leftrightarrow$$

$$\begin{cases} x>4 \\ x \neq \frac{21}{5} \end{cases} \quad C.E = \left(4; \frac{21}{5}\right) \cup \left(\frac{21}{5}; +\infty\right)$$

$$3\log_5(x-4) > \frac{6}{\log_5(x-4)+1}$$

$$\frac{3\log_5(x-4)\left[\log_5(x-4)+1\right]-6}{\log_5(x-4)+1} > 0$$

$$\frac{3\left[\log_5(x-4)\right]^2 + 3\log_5(x-4) - 6}{\log_5(x-4)+1} > 0$$

N) $3\log_5^2(x-4) + 3\log_5(x-4) - 6 > 0$

pongo $\log_5(x-4) = t$

$$3t^2+3t-6>0 \quad t^2+t-2>0$$

$$t_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} \begin{cases} \frac{-1-3}{2} = -\frac{4}{2} = -2 \\ \frac{-1+3}{2} = 1 \end{cases}$$

$t<-2 \cup t>1$

$$\log_5(x-4) < -2 \cup \log_5(x-4) > 1$$

$$\Downarrow$$

$$x-4 < \frac{1}{25} \quad \cup \quad x-4>5$$

$$x < \frac{101}{25} \quad \cup \quad x>9$$

D) $\log_5(x-4)+1>0$

$\log_5(x-4)>-1$

$x-4>\frac{1}{5}$ $\quad x>\frac{21}{5}$ 9

		4		$\frac{101}{25}$		$\frac{21}{5}$		9	x
N	+		+		−	0	−		+
D	−		−		−	0	+		+
S	+		−		$\boxed{+}$	0	−		$\boxed{+}$

m 620 p. 611

$\log_3(2-5x) > 2$

C.E. $\{x \in \mathbb{R} \mid 2-5x > 0\}$

$2-5x > 3^2$

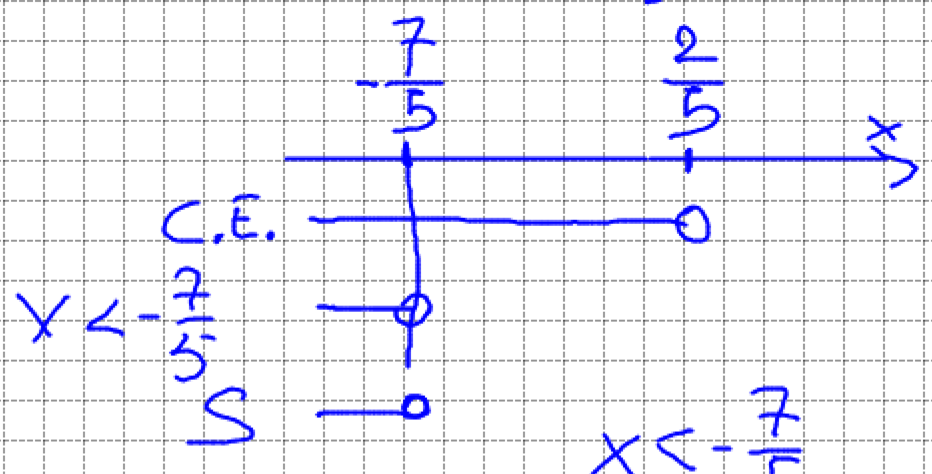
$2-5x > 9$

$-5x > 7 \quad 3x < -7 \quad x < -\frac{7}{5}$

$-5x > -2$

$5x < 2$

$x < \frac{2}{5}$



$x < -\frac{7}{5}$

m 216 p. 589

$3^{2x+2} < \frac{1}{3}$

$3^{2x+2} < 3^{-1}$

$2x+2 < -1$

$2x < -3 \quad x < -\frac{3}{2}$

N 267

$\sqrt[2x]{|4^x - 12|} \geq \sqrt[x]{2}$

$\left[|4^x-12|^{\frac{1}{2x}}\right]^x \geq \left[2^{\frac{1}{x}}\right]^x$

$|4^x - 12|^{\frac{1}{2}} \geq 2$

$|4^x - 12| \geq 4$

$\log_4(4\cdot 3) = \log_4 4 + \log_4 3$

$4^x - 12 \geq 0 \quad 4^x \geq 12 \quad x \geq \log_4 12$

$\begin{cases} x \geq \log_4 3 + 1 \\ 4^x - 12 \geq 4 \end{cases} \cup \begin{cases} x \leq \log_4 3 + 1 \\ -4^x + 12 \geq 4 \end{cases}$

$\begin{cases} x \geq \log_4 3 + 1 \\ 4^x \geq 16 \end{cases} \cup \begin{cases} x \leq \log_4 3 + 1 \\ 4^x \leq 8 \end{cases}$

$\begin{cases} x \geq \log_4 3 + 1 \\ x \geq 2 \end{cases} \cup \begin{cases} x \leq \log_4 3 + 1 \\ x \leq \log_4(4\cdot 2) \end{cases}$