PROPRIETÀ LOGARITMI

$$a^{b_1+b_2} = a^{b_1} \cdot a^{b_2}$$

$b_1 > 0$ e $b_2 > 0$:

1) LOGARITMO DEL PRODOTTO

$$\boxed{\log_a (b_1 \cdot b_2) = \log_a b_1 + \log_a b_2}$$

Dim

$x_1 = \log_a b_1 \qquad x_2 = \log_a b_2$

$a^{x_1} = b_1 \qquad a^{x_2} = b_2$

$$b_1 b_2 = a^{x_1} \cdot a^{x_2}$$

$$b_1 \cdot b_2 = a^{x_1+x_2}$$

$$x_1 + x_2 = \log_a (b_1 \cdot b_2)$$

$$\log_a b_1 + \log_a b_2 = \log_a (b_1 \cdot b_2)$$

2) LOGARITMO DEL QUOZIENTE

$$\boxed{\log_a \left(\frac{b_1}{b_2}\right) = \log_a b_1 - \log_a b_2}$$

Dim

$x_1 = \log_a b_1 \qquad x_2 = \log_a b_2$

$b_1 = a^{x_1} \qquad b_2 = a^{x_2}$

$$\frac{b_1}{b_2} = \frac{a^{x_1}}{a^{x_2}}$$

$$\frac{b_1}{b_2} = a^{x_1 - x_2}$$

$$x_1 - x_2 = \log_a \left(\frac{b_1}{b_2}\right)$$

$$\log_a b_1 - \log_a b_2 = \log_a \left(\frac{b_1}{b_2}\right)$$

3) LOGARITMO DI UNA POTENZA

Sia $b > 0$

$$\boxed{\log_a (b)^k = k \log_a b}$$

Dim

poniamo $x = \log_a b \Rightarrow a^x = b$ eleviamo ambo i membri alla k: $a^{kx} = b^k$

$\Rightarrow k x = \log_a b^k \Rightarrow k \log_a b = \log_a b^k$

$\log_{10} 100 = ?$

$10^{?} = 100$

$10^{?} = 10^2$

$? = 2$

$\log_{10} 10^2 =$

$2 \log_{10} 10 = 2 \cdot 1$

$\log_2 64 = \log_2 2^6 =$

$= 6$

$\log_2 2 = ? \qquad 2^{?} = 2$

$? = 1$

4) $\log_a 1 = 0$, $\log_a a = 1$

5) CAMBIAMENTO DI BASE

ESEMPIO

$$\log_{13} 25 = \frac{\log 25}{\log 13}$$

$$\boxed{\log_a b = \frac{\log_c b}{\log_c a}}$$

Dim

Poniamo $x = \log_a b$ $\quad y = \log_c a$

$a^x = b \quad c^y = a$

$(c^y)^x = b \iff c^{xy} = b$

$xy = \log_c b$

$\log_a b \cdot \log_c a = \log_c b$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

OSSERVA!!

cambia la base in a

$$\log_{\frac{1}{a}} b = \frac{\log_a b}{\log_a \frac{1}{a}} = \frac{\log_a b}{\log_a (a)^{-1}} = \frac{\log_a b}{-1} =$$

$$= -\log_a b$$

$$\boxed{\log_{\frac{1}{a}} b = -\log_a b}$$