ESPONENZIALI

$y = x$ $\quad y = x^2$ $\quad y = x^n$ POTENZE

$$\boxed{y = a^x}$$ funzione esponenziale (fissate alcune condizioni su a)

REGOLE POTENZE

1. $a^0 = 1$
2. $a^1 = a$, $a^2 = a \cdot a$, ..., $a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n\text{-volte}}$
3. $a^{-n} = \frac{1}{a^n}$
4. $a^{n+m} = a^n \cdot a^m$
5. $(a^n)^m = a^{n \cdot m}$
6. $a^{n-m} = a^n \cdot \frac{1}{a^m} = \frac{a^n}{a^m}$
7. $a^{\frac{1}{n}} = \sqrt[n]{a}$
8. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

FUNZIONE $y = a^x$

$y = a^x$

1. $0 < a < 1$
2. $a > 1$

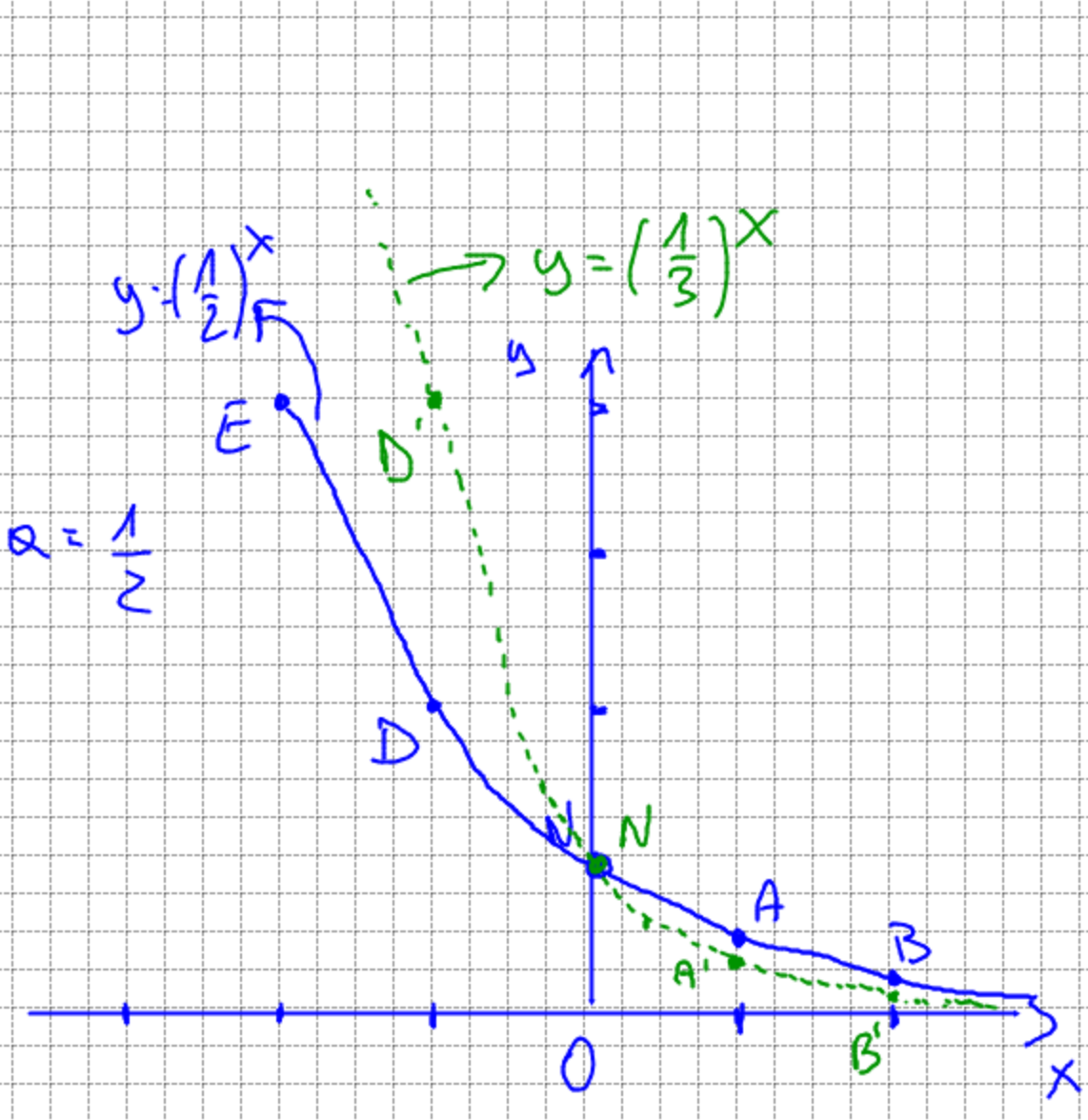
1. $\boxed{0 < a < 1}$ scegliamo $a = \frac{1}{2}$

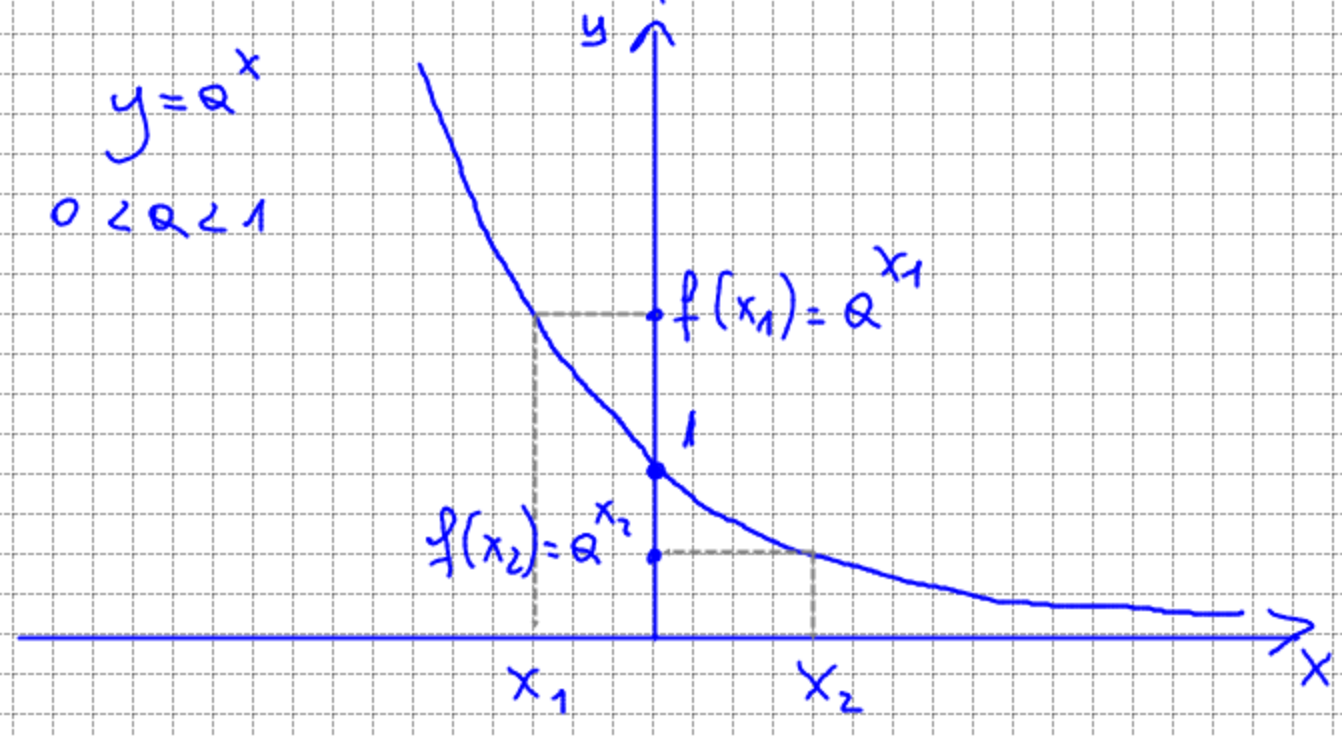
$y = \left(\frac{1}{2}\right)^x$

	x	y
N	0	1
A	1	1/2
B	2	1/4
C	3	1/8
D	-1	2
E	-2	4
F	-3	8

$y = \left(\frac{1}{3}\right)^x$

	x	y
N	0	1
A'	1	1/3
B'	2	1/9
C'	-1	3
D'	-2	9





- $y > 0$
- $y = a^x$ passano tutte per $(0;1)$
- $y = a^x$ è decrescente: $\forall\, x_1, x_2 \in \mathbb{R}$ con $x_1 < x_2 \iff a^{x_1} > a^{x_2}$ (*)

$\left(\frac{1}{3}\right)^{x+1} < \left(\frac{1}{3}\right)^{3x}$ (•) $\Leftrightarrow$ $x+1 > 3x$ $\quad 3x-x<1$

$2x<1 \quad x<\frac{1}{2}$

$x_1 > x_2$ $\Leftrightarrow$ decrescenza

$a^{x_1} < a^{x_2}$ $0<a<1$

$0<a<1 \quad a^{x_1} < a^{x_2} \Leftrightarrow x_1 > x_2$

$y = \left(\frac{1}{3}\right)^{x+1}$ $\quad y = \left(\frac{1}{3}\right)^{x} \cdot \left(\frac{1}{3}\right)^{1}$ $\quad y = \frac{1}{3}\left(\frac{1}{3}\right)^{x}$

$y = \frac{1}{3} \cdot \frac{1}{3^x}$ per $x \to +\infty$ $\quad \frac{1}{3^x} \to 0$

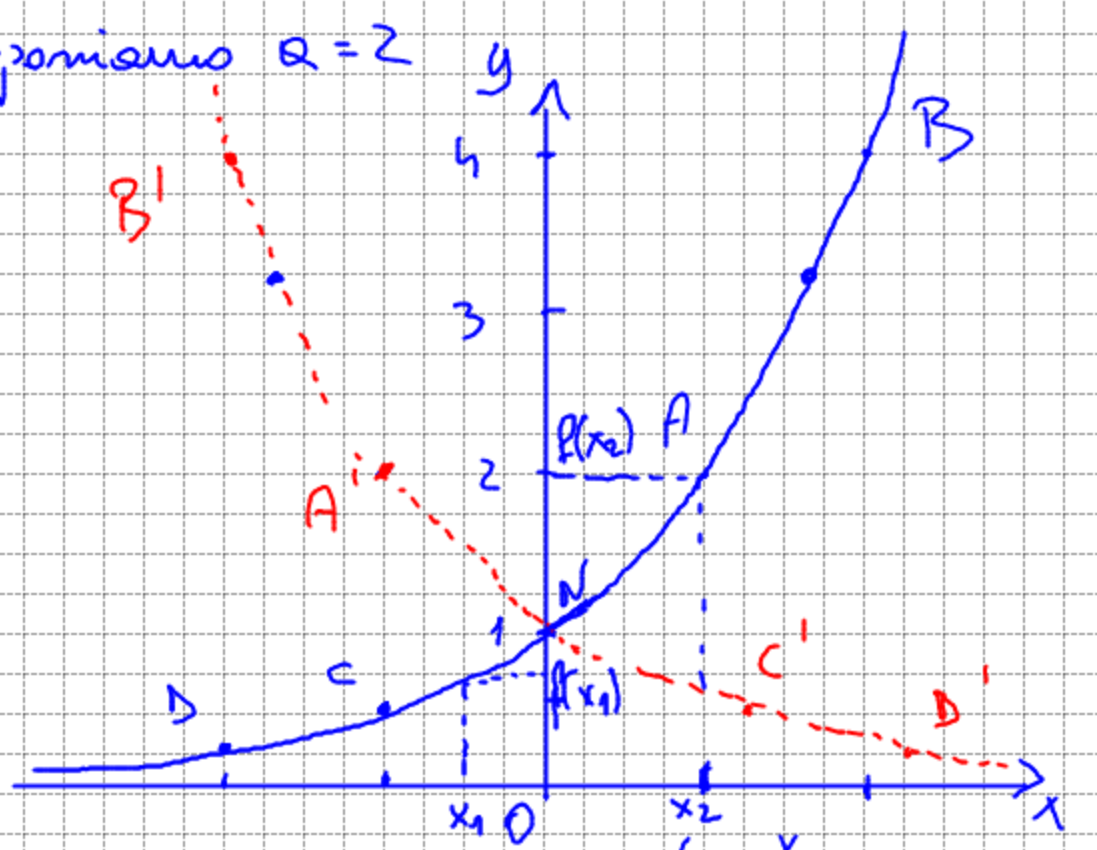
2. $\boxed{a>1}$ $y = a^x$ supponiamo $a=2$

$y = 2^x$

	x	y
N	0	1
A	1	2
B	2	4
C	-1	$\frac{1}{2}$
D	-2	$\frac{1}{4}$

$2^{-1} = \frac{1}{2^1} = \left(\frac{1}{2}\right)^1$

$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$



Questa curva è la curva simmetrica di $y = \left(\frac{1}{2}\right)^x$ rispetto all'asse y $(x=0)$

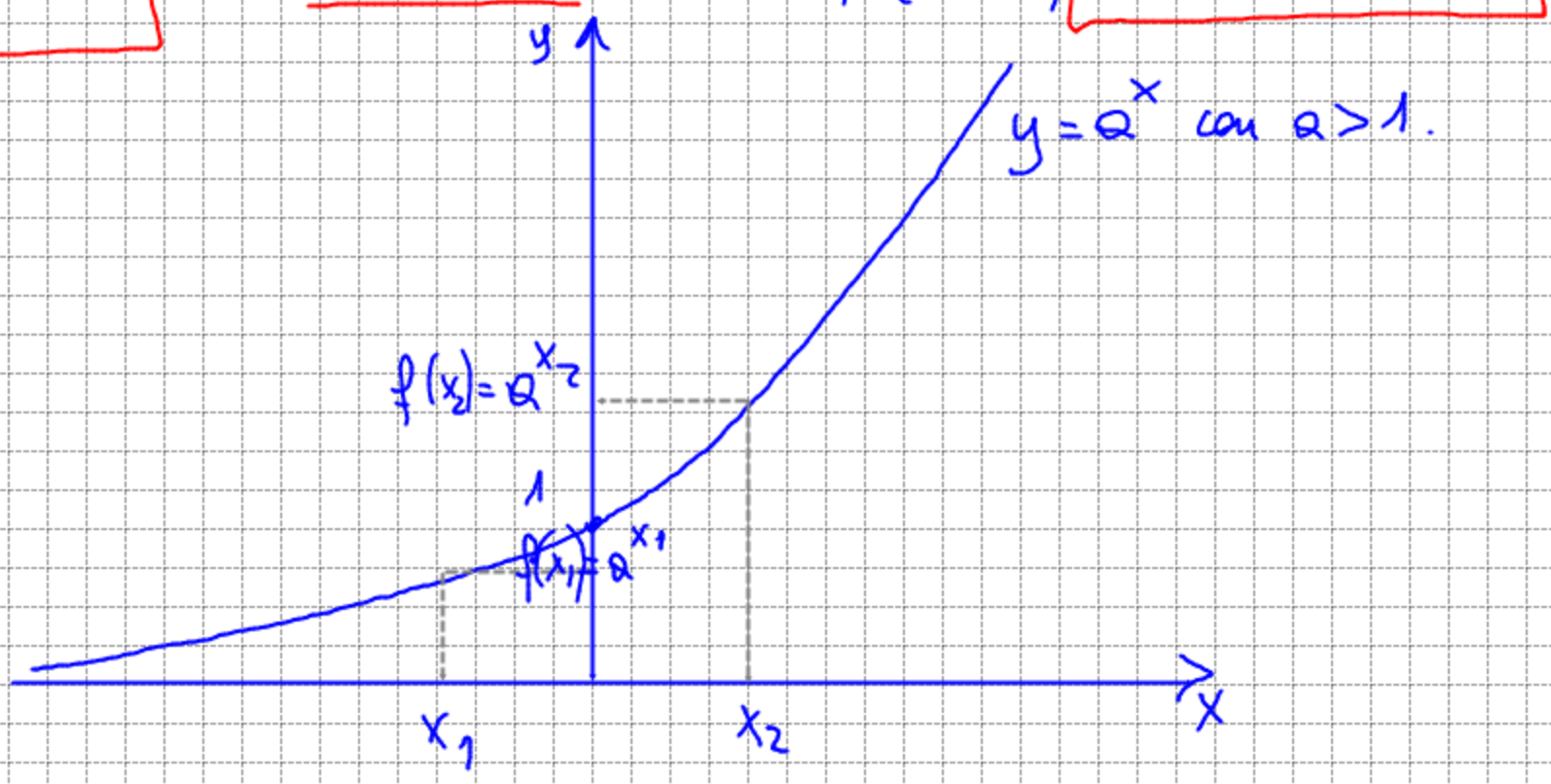
SIMMETRIA S $\begin{cases} y = y' \\ x = -x' \end{cases}$

$y = \left(\frac{1}{2}\right)^x \xrightarrow{S} y' = \left(\frac{1}{2}\right)^{-x'}$

$y' = 2^{x'}$

PROPRIETÀ $(a>1)$

- $y>0$
- $y = a^x$ passano tutte per $(0;1)$
- $\boxed{y = a^x \quad a>1}$ è crescente se $\forall x_1, x_2 \in \mathbb{R}$, $\boxed{x_1 < x_2 \Rightarrow a^{x_1} < a^{x_2}}$



ESEMPIO

$e = 2{,}718281\ldots\ldots$

$e^x - e^{3x-3} < 0$

$e^x < e^{3x-3} \Leftrightarrow x < 3x-3 \quad 3x-x>3 \quad 2x>3 \quad x>\frac{3}{2}$