

INTEGRAZIONE DELLE FUNZIONI RAZIONALI

$$f(x) = \frac{N(x)}{D(x)}$$

$$\boxed{\begin{array}{l|l} N(x) & D(x) \\ R(x) & Q(x) \end{array}}$$

$$f(x) = \frac{N(x)}{D(x)}$$

$$f(x) = Q(x) + \frac{R(x)}{D(x)}$$

$$\int f(x) dx = \int \frac{N(x)}{D(x)} dx = \int Q(x) dx + \int \frac{R(x)}{D(x)} dx$$

1° caso:

$$\int \frac{k}{ax+b} dx$$

$N(x)$ ha grado 0
 $D(x)$ ha grado 1

$$= \frac{k}{a} \int \frac{1}{ax+b} d(ax+b) = \frac{k}{a} \ln|ax+b| + c$$

ESEMPIO

$$\int \frac{1-x^2+3x^4}{2-x} dx =$$

$$\begin{array}{r|l} N(x) & D(x) \\ \hline 3x^4 + 0x^3 - x^2 + 0x + 1 & -x + 2 \\ -3x^4 + 6x^3 & \hline // & 6x^3 - x^2 \\ -6x^3 + 12x^2 & \hline // & 11x^2 + 0x \\ -11x^2 + 22x & \hline // & 22x + 1 \\ -22x + 44 & \hline // & 45 = R(x) = R \end{array}$$

$$\int \frac{3x^4 - x^2 + 1}{-x+2} dx = \int (-3x^3 - 6x^2 - 11x - 22) dx + \int \frac{45}{2-x} dx$$

$$= -\frac{3}{4}x^4 - 2x^3 - \frac{11}{2}x^2 - 22x - 45 \ln|2-x| + c.$$

2° CASO:

$$\int \frac{g(x)+e}{ax^2+bx+c} dx \begin{cases} \rightarrow (1) \Delta > 0 \\ \rightarrow (2) \Delta = 0 \\ \rightarrow (3) \Delta < 0 \end{cases}$$

(1) $\Delta > 0$

ESEMPIO

$$\int \frac{2x-1}{x^2-5x+6} dx = (..) \quad x^2-5x+6 = (x-3)(x-2)$$

$$\frac{2x-1}{x^2-5x+6} = \frac{A}{(x-3)} + \frac{B}{(x-2)}$$
$$\frac{(2)x - 1}{(x-3)(x-2)} = \frac{(A+B)x + (-2A-3B)}{(x-3)(x-2)}$$
$$\begin{cases} A+B=2 \\ -2A-3B=-1 \end{cases}$$

$$\begin{cases} A=2-B \\ 2(2-B)+3B=1 \end{cases} \begin{cases} A=2-B \\ 4+B=1 \end{cases} \begin{cases} A=5 \\ B=-3 \end{cases}$$

$$= \int \frac{5}{(x-3)} dx - \int \frac{3}{(x-2)} dx = 5 \ln|x-3| - 3 \ln|x-2| + C$$

(2) $\Delta = 0 \quad ax^2+bx+c = a(x-d)^2$

ESEMPIO

$$\int \frac{x+1}{x^2-10x+25} dx = (..) \quad D(x^2-10x+25) = 2x-10$$

$$(..) = \frac{1}{2} \int \frac{2x + 2 - 12 + 12}{x^2-10x+25} dx = \frac{1}{2} \int \frac{2x-10}{x^2-10x+25} dx +$$

$$+ \int \frac{12}{x^2-10x+25} dx = \frac{1}{2} \left[\ln|x^2-10x+25| +$$

$$+ 12 \int \frac{1}{(x-5)^2} dx \right] = \frac{1}{2} \ln|x^2-10x+25| +$$

$$+ \frac{12(-1)}{2(x-5)} + C.$$