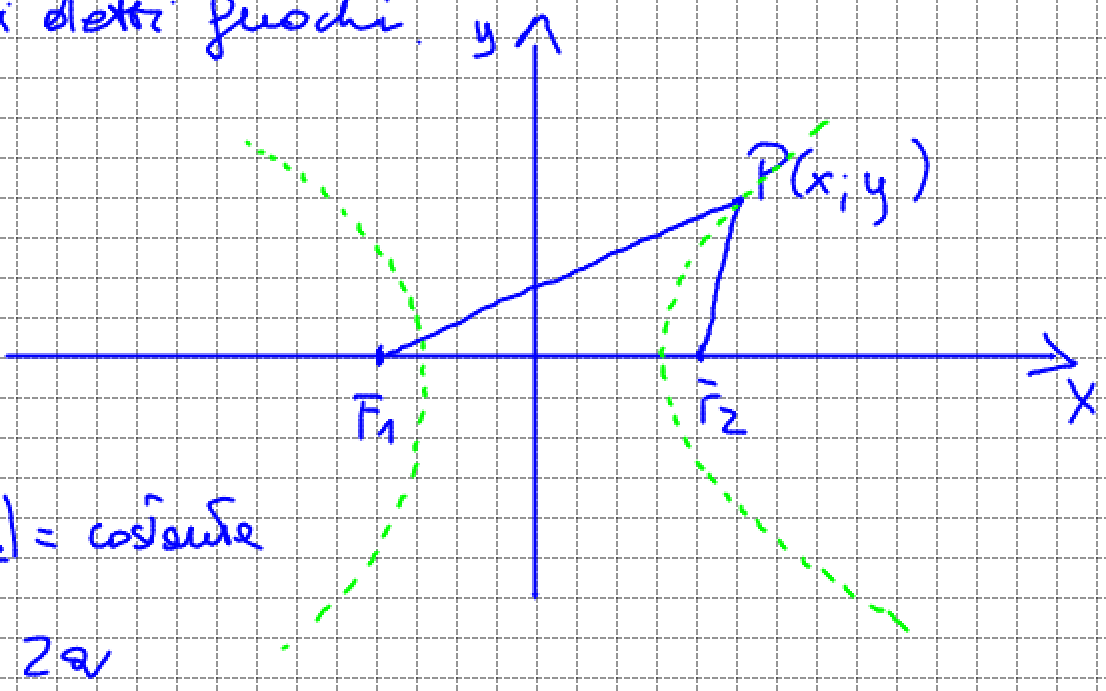
IPERBOLE

Def: L'iperbole è il luogo geometrico dei punti del piano $P(x;y)$ per cui è costante la differenza delle distanze da due punti fissi detti fuochi.



$F_1(-c;0)$
$F_2(c;0)$

$|\overline{PF_1}-\overline{PF_2}| = \text{costante}$

costante $= 2a$

$$|\overline{PF_1}-\overline{PF_2}| = 2a \qquad 2a < 2c \Rightarrow \boxed{a < c}$$

$$\left|\sqrt{(x+c)^2+(y-0)^2}-\sqrt{(x-c)^2+(y-0)^2}\right| = 2a$$

$$\sqrt{(x+c)^2+y^2}-\sqrt{(x-c)^2+y^2} = \pm 2a$$

$$\sqrt{(x+c)^2+y^2} = \sqrt{(x-c)^2+y^2} \pm 2a$$

$$(x+c)^2+y^2 = (x-c)^2+y^2+4a^2 \pm 4a\sqrt{(x-c)^2+y^2}$$

$$\cancel{x^2}+\cancel{c^2}+2xc+\cancel{y^2} = \cancel{x^2}+\cancel{c^2}-2cx+\cancel{y^2}+4a^2 \pm 4a\sqrt{(x-c)^2+y^2}$$

$$(4cx-4a^2)^2 = \left(\pm 4a\sqrt{(x-c)^2+y^2}\right)^2$$

$$\cancel{16}c^2x^2+\cancel{16}a^4-\cancel{32}a^2cx = \cancel{16}a^2\left[x^2+c^2-2cx+y^2\right]$$

$$c^2x^2+a^4-\cancel{2a^2cx} = a^2x^2+a^2c^2-\cancel{2a^2cx}+a^2y^2$$

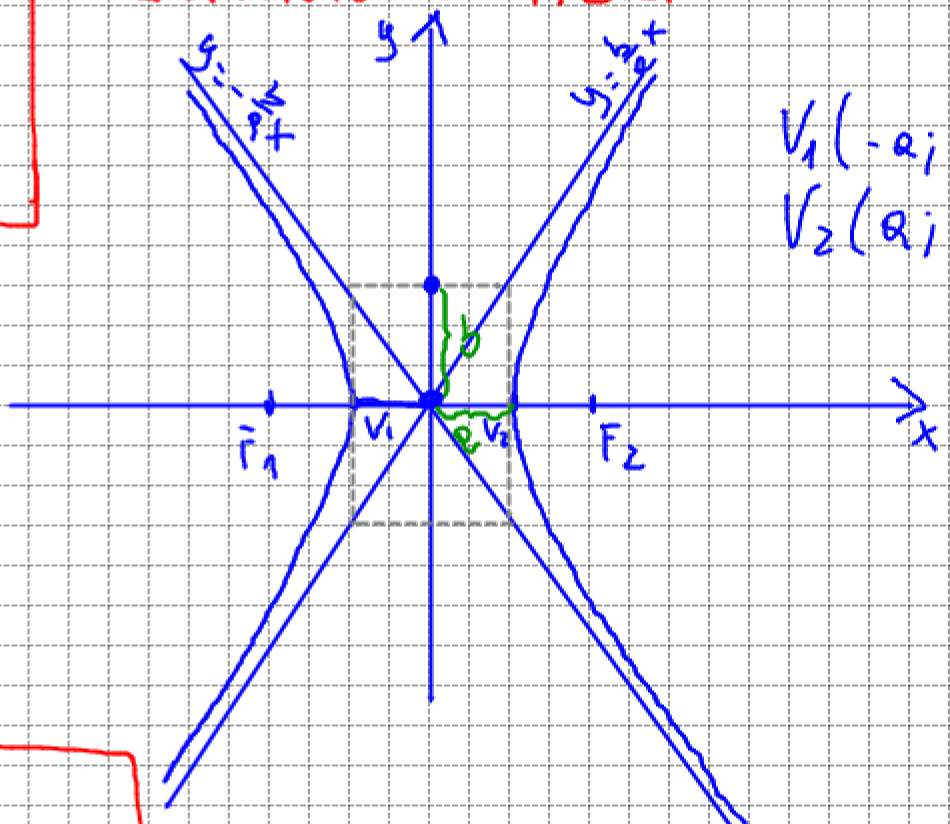
$$x^2(c^2-a^2)-a^2y^2 = a^2c^2-a^4$$

pongo $c^2-a^2 = b^2$

$$x^2b^2-a^2y^2 = a^2b^2$$

$$\boxed{\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1 \qquad c > a \text{ e } c^2-a^2=b^2}$$

EQUAZIONE IPERBOLE



$V_1(-a;0)$
$V_2(a;0)$

$$\frac{x^2}{a^2} = \frac{y^2}{b^2}+1$$

$$x = \pm a\sqrt{\frac{y^2}{b^2}+1}$$

$$\boxed{y = \pm\frac{b}{a}x \quad \text{ASINTOTI}}$$

$$\boxed{e = \frac{c}{a} \qquad e = \frac{\sqrt{a^2+b^2}}{a} > 1 \quad \text{perché} \quad e = \sqrt{\frac{a^2+b^2}{a^2}} = \sqrt{1+\frac{b^2}{a^2}}}$$

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Luogo geometrico: $F_1(-4;0)$; $F_2(4;0)$

$P(x;y) \quad |\overline{PF_1} - \overline{PF_2}| = 2\sqrt{10}$

$$\left|\sqrt{(x+4)^2+(y-0)^2} - \sqrt{(x-4)^2+(y-0)^2}\right| = 2\sqrt{10}$$

$$\left(\sqrt{(x+4)^2+y^2} - \sqrt{(x-4)^2+y^2}\right)^2 = \left(\pm 2\sqrt{10}\right)^2$$

$$(x+4)^2+y^2+(x-4)^2+y^2-2\sqrt{[(x+4)^2+y^2][(x-4)^2+y^2]} = 40$$

$$\underline{x^2}+16+\cancel{8x}+y^2+\underline{x^2}+16\cancel{-8x}+y^2-2\sqrt{[(x+4)^2+y^2][(x-4)^2+y^2]} = 40$$

$$x^2-4+y^2 = \sqrt{[(x+4)^2+y^2][(x-4)^2+y^2]}$$

$$x^4+16+y^4-8x^2+2x^2y^2-8y^2 = (x^2+16+8x+y^2)\cdot(x^2-8x+16+y^2)$$

$$\cancel{x^4}+16+\cancel{y^4}-8x^2+2\cancel{x^2y^2}-8y^2 = \cancel{x^4}\cancel{-8x^3}+\underline{16x^2}+\cancel{x^2y^2}+\underline{16x^2}\cancel{-128x}+256+16y^2+\cancel{8x^3}+$$
$$\underline{-64x^2}\cancel{+128x}+\cancel{8xy^2}+\cancel{x^2y^2}\cancel{-8xy^2}+16y^2+\cancel{y^4}$$

$$16-8x^2-8y^2+32x^2-256-32y^2=0$$

$$-40y^2+24x^2-240=0$$

$$-20y^2+12x^2-120=0$$

$$-5y^2+3x^2-30=0$$

$$\frac{3x^2}{30}-\frac{5y^2}{30}=\frac{30}{30} \qquad \boxed{\frac{x^2}{10}-\frac{y^2}{6}=1} \qquad a=\sqrt{10} \quad b=\sqrt{6}$$

$$y=\pm\sqrt{\frac{\cancel{6}^{3}}{\cancel{10}_{5}}}\,x$$