PROBLEMA

Utilizzando la definizione, trova la derivata della seguente funzione e conferma il risultato con le regole di derivazione.

$$y = \sqrt{x^2+5} \quad \text{in } x = 1$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} =$$

$$= \lim_{h\to 0} \frac{\sqrt{(x+h)^2+5}-\sqrt{x^2+5}}{h} =$$

$$= \lim_{h\to 0} \frac{\sqrt{(1+h)^2+5}-\sqrt{1+5}}{h} =$$

$$= \lim_{h\to 0} \frac{\sqrt{1+h^2+2h+5}-\sqrt{6}}{h} =$$

$$= \lim_{h\to 0} \frac{\sqrt{h^2+2h+6}-\sqrt{6}}{h} =$$

$$= \lim_{h\to 0} \frac{\sqrt{h^2+2h+6}-\sqrt{6}}{h} \cdot \frac{\sqrt{h^2+2h+6}+\sqrt{6}}{\sqrt{h^2+2h+6}+\sqrt{6}} =$$

$$= \lim_{h\to 0} \frac{h^2+2h+\cancel{6}-\cancel{6}}{h\left(\sqrt{h^2+2h+6}+\sqrt{6}\right)} =$$

$$= \lim_{h\to 0} \frac{\cancel{h}(h+2)}{\cancel{h}\left(\sqrt{h^2+2h+6}+\sqrt{6}\right)} = \frac{\cancel{2}}{\cancel{2}\sqrt{6}} = \frac{1}{\sqrt{6}}$$

$$D\left(\sqrt{x^2+5}\right) = D\left(\left(x^2+5\right)^{\frac{1}{2}}\right) = \frac{1}{2}\left(x^2+5\right)^{\frac{1}{2}-1} D\left(x^2+5\right) =$$

$$= \frac{1}{\cancel{2}\sqrt{x^2+5}} \cdot \cancel{2}x = \frac{x}{\sqrt{x^2+5}}$$

$$D\left(\sqrt{x^2+5}\right)\Big|_{x=1} = \frac{1}{\sqrt{6}}$$

PROBLEMA

$$y = -\cos 2x \quad \left(f(x)\right)$$

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{-\cos 2(x+h) - (-\cos 2x)}{h} =$$

$$= \lim_{h \to 0} \frac{-\cos(2x+2h) + \cos 2x}{h} =$$

$$= \lim_{h \to 0} \frac{-\cos 2x \cos 2h + \operatorname{sen} 2x \operatorname{sen} 2h + \cos 2x}{h} =$$

$$= \lim_{\substack{h \to 0 \\ 2h \to 0}} \left[2\cos 2x \underbrace{\frac{-\cos 2h + 1}{2h}}_{\xrightarrow{h \to 0}\ 0} + 2\operatorname{sen} 2x \underbrace{\frac{\operatorname{sen} 2h}{2h}}_{\xrightarrow{h \to 0}\ 1} \right] =$$

$$= 2 \operatorname{sen} 2x$$

$$D\left(-\cos 2x\right) = \operatorname{sen} 2x \cdot \left(2\right) = 2 \operatorname{sen} 2x$$

PROBLEMA

$$y = \frac{2-x}{x \ln x} \qquad y' = \frac{D(2-x) \cdot x \ln x - (2-x) D(x \ln x)}{(x \ln x)^2}$$

$$y' = \frac{-x \ln x - (2-x)\left[D(x) \cdot \ln x + x D(\ln x)\right]}{(x \ln x)^2}$$

$$y' = \frac{-x \ln x - (2-x)\left[\ln x + \cancel{x}^{1} \frac{1}{\cancel{x}}\right]}{(x \ln x)^2}$$

$$y' = \frac{-x \ln x - (2-x)(\ln x + 1)}{x^2 (\ln x)^2}$$

$$y' = \frac{-\cancel{x \ln x} - 2 \ln x - 2 + \cancel{x \ln x} + x}{x^2 (\ln x)^2}$$

$$y' = \frac{x - 2(\ln x + 1)}{x^2 (\ln x)^2}$$

PROBLEMA

1) $f(x) = \left(e^{x^2}+3\right)^2$

$f'(x) = 2\left(e^{x^2}+3\right)^{2-1} D\left(e^{x^2}+3\right) =$

$= 2\left(e^{x^2}+3\right)\left(2x\,e^{x^2}\right) = 4x\,e^{x^2}\left(e^{x^2}+3\right)$

2) $f(x) = \dfrac{1}{x^{\cos x}}$ $\qquad f(x) = x^{-\cos x}$ $\qquad e^{\ln x^{-\cos x}}$

$f(x) = e^{-\cos x \cdot \ln x}$

$f'(x) = e^{-\cos x \ln x} D\left(-\cos x \ln x\right) =$

$= e^{-\cos x \ln x}\left[\operatorname{sen} x \ln x - \dfrac{\cos x}{x}\right]$

3) $D\left(\sqrt{\dfrac{x+3}{2-x}}\right) = \dfrac{1}{2\sqrt{\dfrac{x+3}{2-x}}} D\left(\dfrac{x+3}{2-x}\right) =$

$= \dfrac{\sqrt{2-x}}{2\sqrt{x+3}}\left(\dfrac{1(2-x)+1(x+3)}{(2-x)^2}\right)$

PROBLEMA

Dimostra che $y = |x-x^2|$ non è derivabile in $x=1$.

$x - x^2 \geq 0 \qquad x(1-x) \geq 0$ $\quad x \geq 0$; $x \leq 1$

		0		1	
$x \geq 0$	−		+		+
$x \leq 1$	+		+		−
	−		+		−

$f(x) = \begin{cases} -x+x^2 & \text{per } x<0 \cup x\geq 1 \\ x-x^2 & 0 \leq x < 1 \end{cases}$

$\left.\begin{array}{l} f_+(1) = f(x)\big|_{x=1} = -1+(1)^2 = 0 \\ f_-(1) = \lim\limits_{x\to 1^-}(x-x^2) = 0 \end{array}\right\}$ $f(x)$ è continua in $x=1$

$f'(x) = \begin{cases} -1+2x & x<0 \cup x\geq 1 \\ 1-2x & 0\leq x<1 \end{cases}$

$\left.\begin{array}{l} f'_+(1) = -1+2(1) = 1 \\ f'_-(1) = \lim\limits_{x\to 1^-}(1-2x) = -1 \end{array}\right\}$ $f(x)$ non è derivabile in $x=1$