FASCI DI CIRCONFERENZE

Date $\mathcal{C}_1, \mathcal{C}_2$ due circonferenze

$$\mathcal{C}_1 : x^2+y^2+a_1x+b_1y+c_1=0$$
$$\mathcal{C}_2 : x^2+y^2+a_2x+b_2y+c_2=0$$

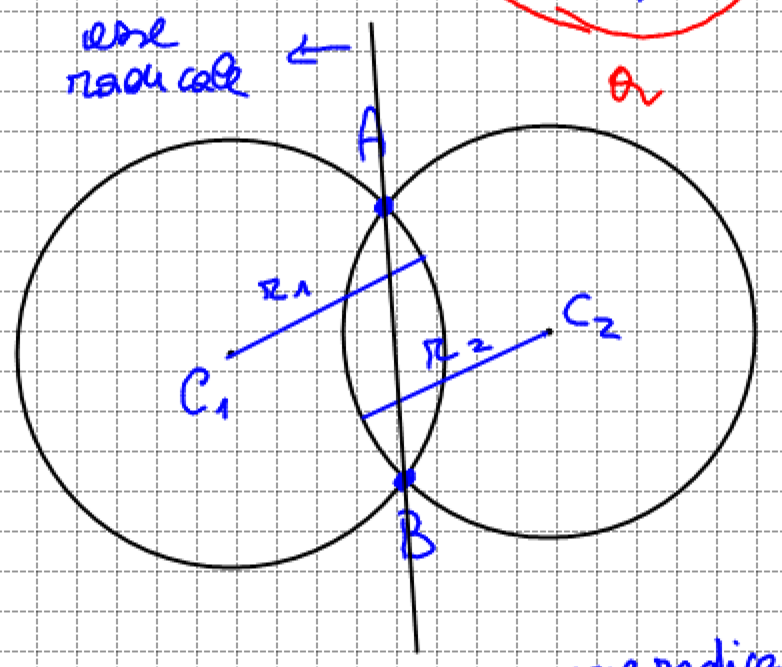
λ, μ due parametri reali, si chiama fascio di circonferenze l'equazione $\boxed{\lambda\mathcal{C}_1+\mu\mathcal{C}_2=0}$ (combinazione lineare delle due circonferenze)

EQ. FASCIO DI CIRCONFERENZE
$$\boxed{\lambda\left(x^2+y^2+a_1x+b_1y+c_1\right)+\mu\left(x^2+y^2+a_2x+b_2y+c_2\right)=0}$$

$$(\lambda+\mu)x^2+(\lambda+\mu)y^2+(\lambda a_1+\mu a_2)x+(\lambda b_1+\mu b_2)y+\lambda c_1+\mu c_2=0$$

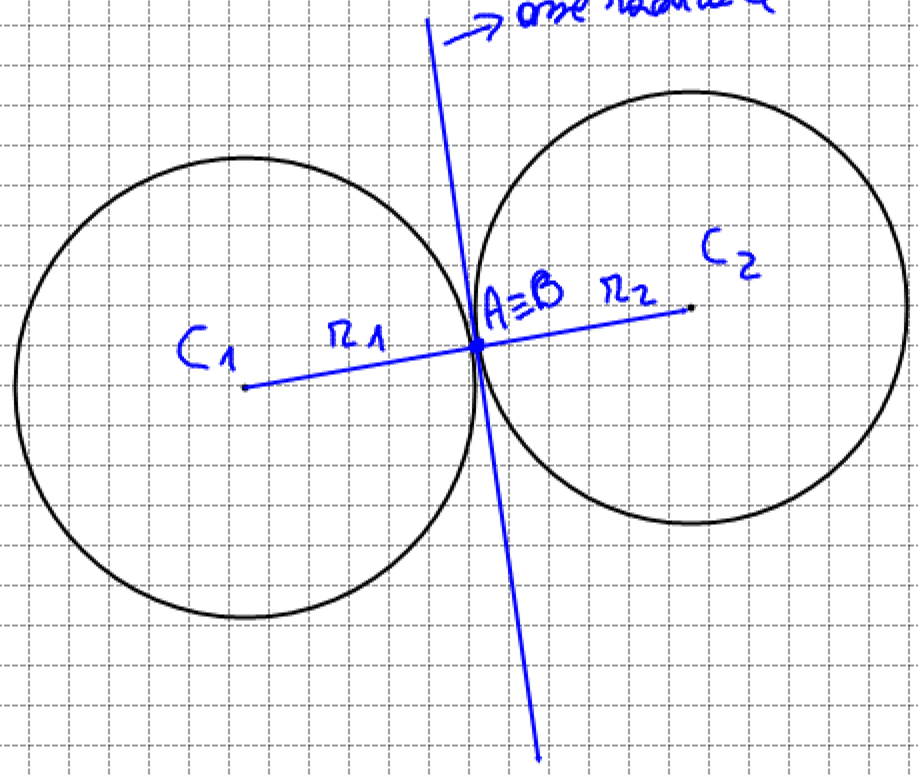
posto $\lambda+\mu\neq 0$

$$\mathcal{F}_{\mathcal{C}} : \quad x^2+y^2+\underbrace{\frac{\lambda a_1+\mu a_2}{\lambda+\mu}}_{a}x+\underbrace{\frac{\lambda b_1+\mu b_2}{\lambda+\mu}}_{b}y+\underbrace{\frac{\lambda c_1+\mu c_2}{\lambda+\mu}}_{c}=0$$

1) 

A, B : punti base

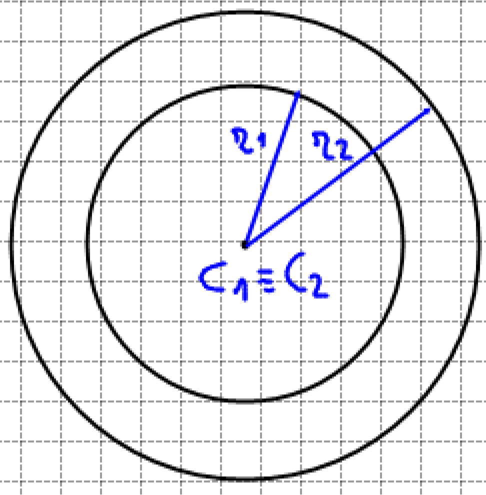
r_{AB} : asse radicale (secante)

2) 

$r_{B\equiv A} \perp r_{C_1C_2}$

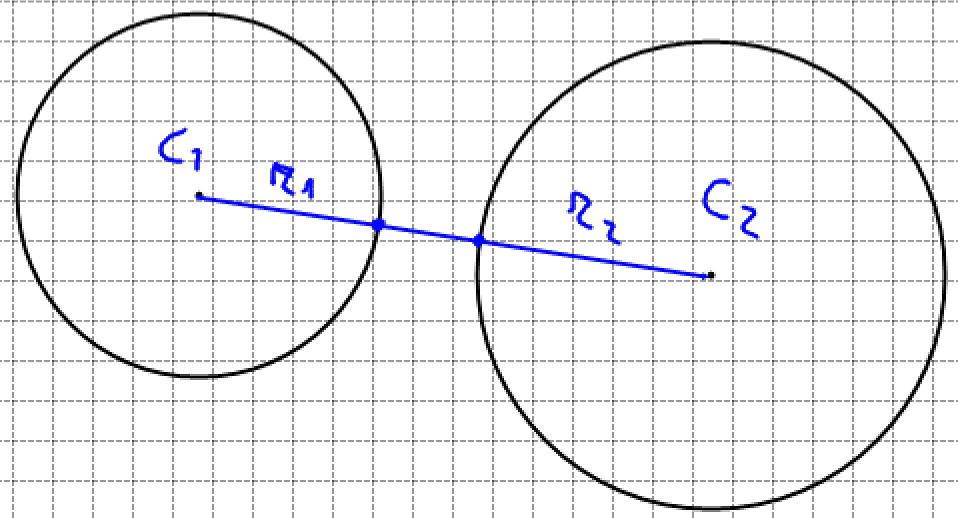
$r_{B\equiv A}$: asse radicale (tangente)

$A\equiv B$: punto base.

3) 

$\mathcal{C}_1\equiv\mathcal{C}_2\equiv\left(-\frac{a}{2}; -\frac{b}{2}\right)$

$d(C_1; C_2) > r_1+r_2$

4) 

ESEMPIO

1) $x^2+y^2-1=0 : \mathcal{C}_1$ $\qquad$ $\mathcal{C}_2 : x^2+y^2-2x=0$

$\lambda \mathcal{C}_1 + \mu \mathcal{C}_2 = 0$

$$\boxed{\lambda(x^2+y^2-1)+\mu(x^2+y^2-2x)=0}$$

$\lambda \neq 0$ divido per λ e pongo $k=\frac{\mu}{\lambda}$

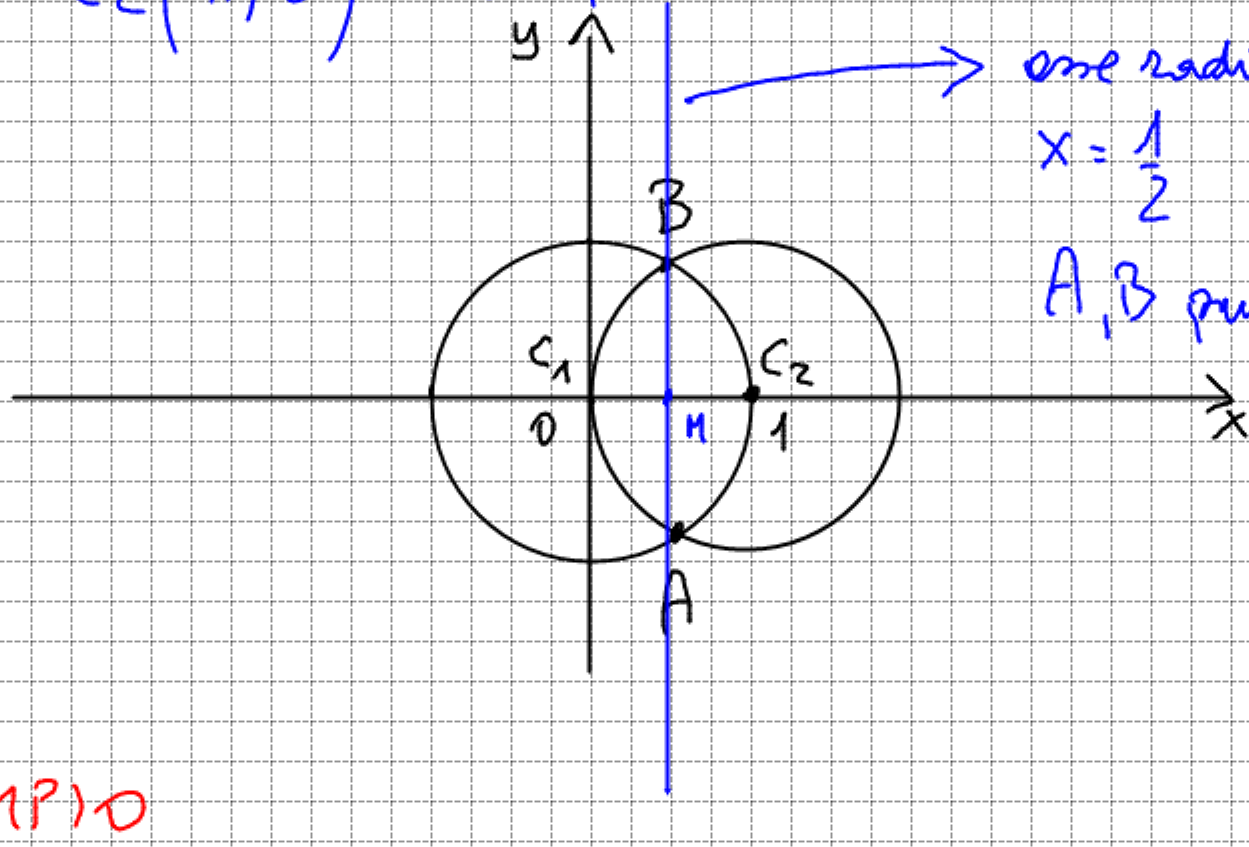
$$\boxed{x^2+y^2-1+k(x^2+y^2-2x)=0 \ \cup \ x^2+y^2-2x=0}$$

$$\begin{cases} x^2+y^2=1 \\ x^2+y^2-2x=0 \end{cases} \quad \begin{cases} 2x=1 \\ x^2+y^2=1 \end{cases} \quad \begin{cases} x=\frac{1}{2} \\ y^2=1-\frac{1}{4} \end{cases} \quad \begin{cases} x=\frac{1}{2} \\ y=\pm\frac{\sqrt{3}}{2} \end{cases}$$

$A\left(\frac{1}{2}; -\frac{\sqrt{3}}{2}\right)$ $\quad$ $B\left(\frac{1}{2}; +\frac{\sqrt{3}}{2}\right)$

$\mathcal{C}_1 : \ C_1(0;0) \quad r=1$

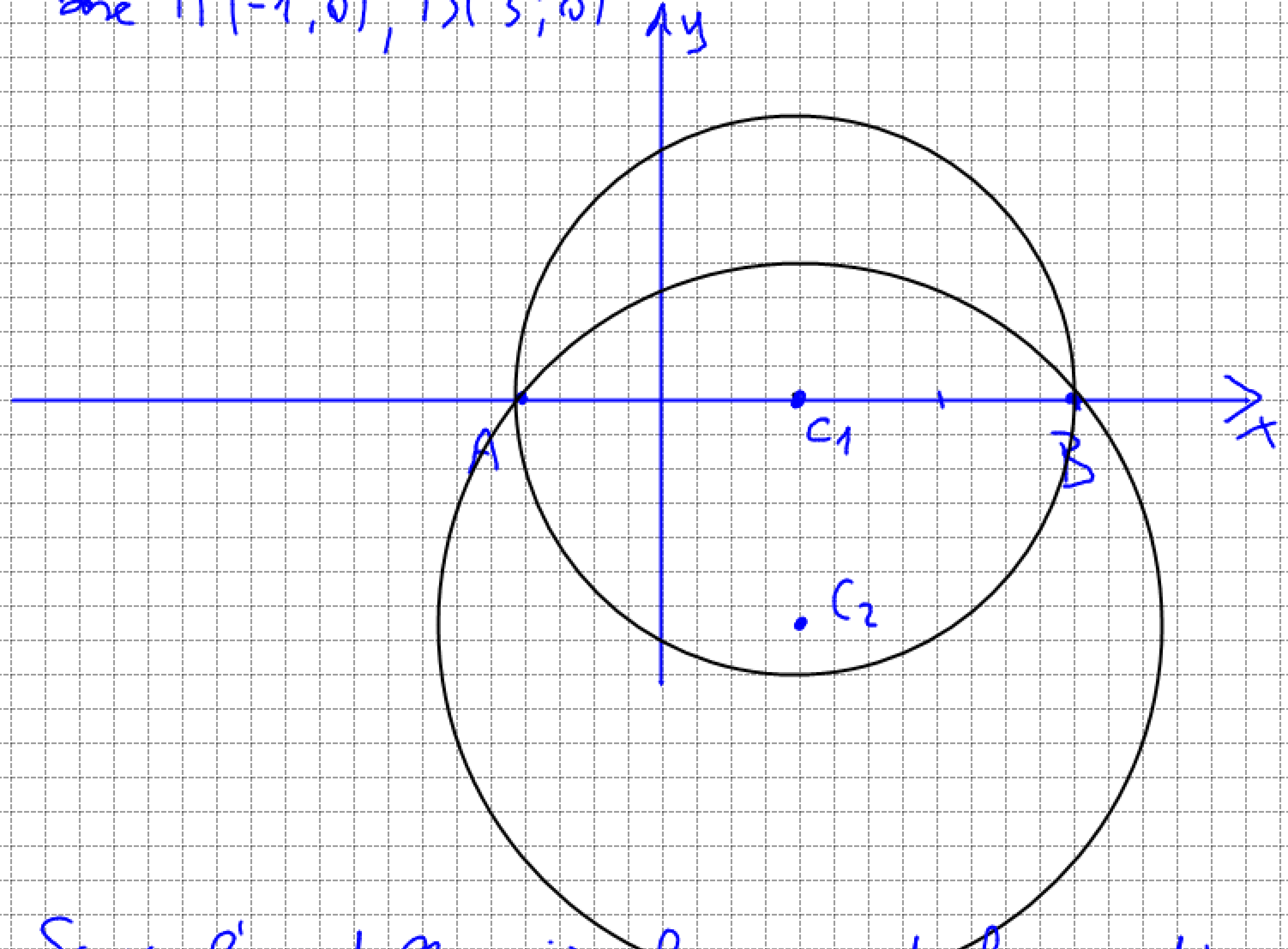
$\mathcal{C}_2 : \ C_2(1;0) \quad r=1$



asse radicale:
$x=\frac{1}{2}$
A, B punti base.

ESEMPIO

Scrivere l'eq. del fascio di circonferenze che ha come punti base $A(-1;0)$, $B(3;0)$



Scrivo l'eq. della circonferenza che ha per diametro $\overline{AB}$:

$C_1\left(\frac{x_A+x_B}{2}; \frac{y_A+y_B}{2}\right) \Rightarrow \mathcal{C}_1(1;0) \qquad r=2$

$(x-1)^2+(y-0)^2=4 \qquad x^2+y^2-2x-3=0 \ \mathcal{C}_1$

considero un'altra circonferenza che passa per A, per B e ha raggio $r=3$ (inventato da me)

$$x^2+y^2+ax+by+c=0$$

$A(-1;0) \quad B(3;0) \quad r=3 \quad \sqrt{\left(-\frac{a}{2}\right)^2+\left(-\frac{b}{2}\right)^2-c}=3$

$$\begin{array}{l} A \\ B \\ r=3 \end{array}\begin{cases} 1+\cancel{0}-a+\cancel{0b}+c=0 \\ 9+\cancel{0}+3a+\cancel{0b}+c=0 \\ +\frac{a^2}{4}+\frac{b^2}{4}-c=9 \end{cases} \begin{cases} 1-a+c=0 \\ 9+3a+c=0 \\ \frac{a^2}{4}+\frac{b^2}{4}-c=9 \end{cases}$$

$$\begin{array}{r} R_2-R_1 \\ R_1 \\ R_3 \end{array}\begin{cases} 8+4a=0 \longrightarrow a=-2 \\ c=a-1 \\ 1+\frac{b^2}{4}+2+1=9 \end{cases} \begin{cases} a=-2 \\ c=-3 \\ b=\pm\sqrt{20} \end{cases}$$

$\mathcal{C}_2 \quad x^2+y^2-2x+\sqrt{20}\,y-3=0$

$\mathcal{C}_3 \quad x^2+y^2-2x-\sqrt{20}\,y-3=0$

$$\lambda\left(x^2+y^2-2x-3\right)+\mu\left(x^2+y^2-2x+\sqrt{20}\,y-3\right)=0$$