Q5

Determina equazione asintoto obliquo di

$$f(x) = \frac{x}{2^{\frac{1}{x}}+1}$$

$$D_{f(x)} = \left\{ x \in \mathbb{R} \,/\, \begin{cases} x \neq 0 \\ 2^{\frac{1}{x}}+1 \neq 0 \end{cases} \right\} \Rightarrow (0; +\infty)$$

Asintoto obliquo: $y = mx + q$ con

$$m = \lim_{x \to \infty} \frac{f(x)}{x} \qquad q = \lim_{x \to \infty} \left(f(x) - mx \right)$$

$$m = \lim_{x \to \infty} \frac{\cancel{x}}{\cancel{x}\left(2^{\frac{1}{x}}+1\right)} = \frac{1}{2}$$

$$q = \lim_{x \to \infty} \left(\frac{x}{2^{\frac{1}{x}}+1} - \frac{1}{2}x \right) =$$

$$= \lim_{x \to \infty} \left(\frac{2x - x\left(2^{\frac{1}{x}}+1\right)}{2\left(2^{\frac{1}{x}}+1\right)} \right) =$$

$$= \lim_{x \to \infty} \left(\frac{2x - x \cdot 2^{\frac{1}{x}} - x}{2\left(2^{\frac{1}{x}}+1\right)} \right) = \lim_{x \to \infty} \frac{x - x\,2^{\frac{1}{x}}}{2\left(2^{\frac{1}{x}}+1\right)} =$$

$$= \lim_{x \to \infty} \frac{x\left(1 - 2^{\frac{1}{x}}\right)}{2\left(2^{\frac{1}{x}}+1\right)} = \lim_{x \to \infty} \frac{-1}{2\left(2^{\frac{1}{x}}+1\right)} \cdot \frac{2^{\frac{1}{x}}-1}{\frac{1}{x}} =$$

$$= -\frac{1}{4} \ln 2$$

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a$$

$$\lim_{x \to \infty} \frac{2^{\frac{1}{x}} - 1}{\frac{1}{x}}$$

pongo $\frac{1}{x} = t$

$x \to \infty \quad t \to 0$

$$\parallel$$

$$\lim_{t \to 0} \frac{2^t - 1}{t} = \ln 2$$

$$y = \frac{1}{2}x - \frac{1}{4} \ln 2$$

Q6

$\binom{n}{k}$ $\quad n \geq k \geq 0$

$$CE \begin{cases} x \geq 5 \\ x+2 \geq 5 \end{cases} \quad x \geq 5$$

$$6\binom{x}{5} = \binom{x+2}{5}$$

$$6\,\frac{x!}{\cancel{5!}\,(x-5)!} = \frac{(x+2)!}{\cancel{5!}\,(x+2-5)!}$$

$$\frac{6\,\cancel{x!}}{(x-5)!} = \frac{(x+2)(x+1)\,\cancel{x!}}{(x-3)!} \qquad 0! = 1$$

$$\frac{6}{\cancel{(x-5)!}} = \frac{(x+2)(x+1)}{(x-3)(x-4)\cancel{(x-5)!}}$$

$$6(x-3)(x-4) = (x+2)(x+1)$$

$\cancel{x=2}$ $\quad x = 7$

Q7

$$f(x) = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$$

$D_{f(x)} = ?$

asintoto verticale?

$$D_{f(x)} = \left\{ x \in \mathbb{R} \,/\, x > 0 \right\} = (0; +\infty)$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left[\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 \right] = 0$$

$y = f(x)$ non ammette asintoti verticali.

Q8

$f(x): y = \operatorname{sen} 2x$ $\qquad$ $y = \operatorname{sen} nx \quad n \in \mathbb{N}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\operatorname{sen}2(x+h)-\operatorname{sen}2x}{h}$$

$$= \lim_{h\to 0} \frac{\operatorname{sen}(2x+2h)-\operatorname{sen}2x}{h} =$$

$$= \lim_{h\to 0} \frac{2 \operatorname{sen} \frac{\cancel{2x}+2h-\cancel{2x}}{2} \cos \frac{2x+2h+2x}{2}}{h} =$$

$$= \lim_{h\to 0} \frac{2 \operatorname{sen} h \cdot \cos(2x+h)}{h} =$$

$$= 2\cos 2x$$

$$\operatorname{sen} p - \operatorname{sen} q = 2 \operatorname{sen} \frac{p-q}{2} \cos \frac{p+q}{2}$$

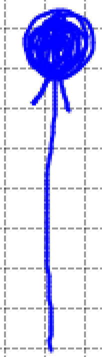
se $g(x) = \operatorname{sen} nx \quad n \in \mathbb{N}$

$$g'(x) = n \cos nx$$

Q 9

$h(t) = 40t - 2t^2$ (metri)

$v(t) = ?$ quota max = ?



$s(t) = h(t)$ $\qquad v(t) = \dfrac{ds(t)}{dt}$

$v(t) = h'(t) = 40 - 4t$

in h_{max} $\qquad v(t) = 0 \qquad 40 - 4t = 0 \qquad t = 10$

$h_{max} = h(10) = 200\ m$

$$f'(x) = \frac{d f(x)}{d x} = D\big(f(x)\big)$$

Q 10

$$y = \frac{|x-2|}{x-2} \ln(x-1) \quad \begin{cases} \text{se } x \geq 2 & \ln(x-1) \\ \text{se } x < 2 & -\ln(x-1) \end{cases}$$

$$D_{f(x)} = \left\{ x \in \mathbb{R} \,/\, \begin{cases} x-2 \neq 0 \\ x-1 > 0 \end{cases} \right\} = \left\{ x \in \mathbb{R} \,/\, \begin{cases} x \neq 2 \\ x > 1 \end{cases} \right\} =$$

$$= \left\{ x \in \mathbb{R} \,/\, (1;2) \cup (2;+\infty) \right\} =$$

$$= (1;2) \cup (2;+\infty)$$

$\lim_{x \to 1^+} f(x) = +\infty$ $\quad x = 1$ Asintoto verticale destro.

discontinuità di II specie in $x = 1$.

$\lim_{x \to 2^-} f(x) = 0^+$ $\quad \lim_{x \to 2^+} f(x) = 0^+$

$x = 2$ $f(x)$ è discontinua III specie.

$$f(x) = \begin{cases} \frac{|x-2|}{x-2} \ln(x-1) & x \neq 2 \\ 0 & x = 2 \end{cases}$$