

$$f(x) = \log(x+3)$$

$$x_0 = 0$$

Calcolare la derivata:

$$f(0+h) = \log(0+h+3)$$

$$\lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{\log(0+3+h) - \log(0+3)}{h} = \lim_{h \rightarrow 0} \frac{\log(3+h) - \log 3}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \log\left(1 + \frac{h}{3}\right) = \lim_{h \rightarrow 0} \log\left(1 + \frac{h}{3}\right)^{\frac{1}{h}}$$

$$\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\frac{1}{x} = \frac{h}{3} \quad h \rightarrow 0 \quad x \rightarrow +\infty$$

$$\frac{1}{h} = \frac{3}{x}$$

$$(*) = \lim_{x \rightarrow +\infty} \log\left(1 + \frac{1}{x}\right)^{\frac{x}{3}} = \lim_{x \rightarrow +\infty} \frac{1}{3} \log\left(1 + \frac{1}{x}\right)^x$$

$$= \frac{1}{3} \log e = \frac{1}{3} \cdot 1$$