PUNTO DI FLESSO A TANGENTE VERTICALE

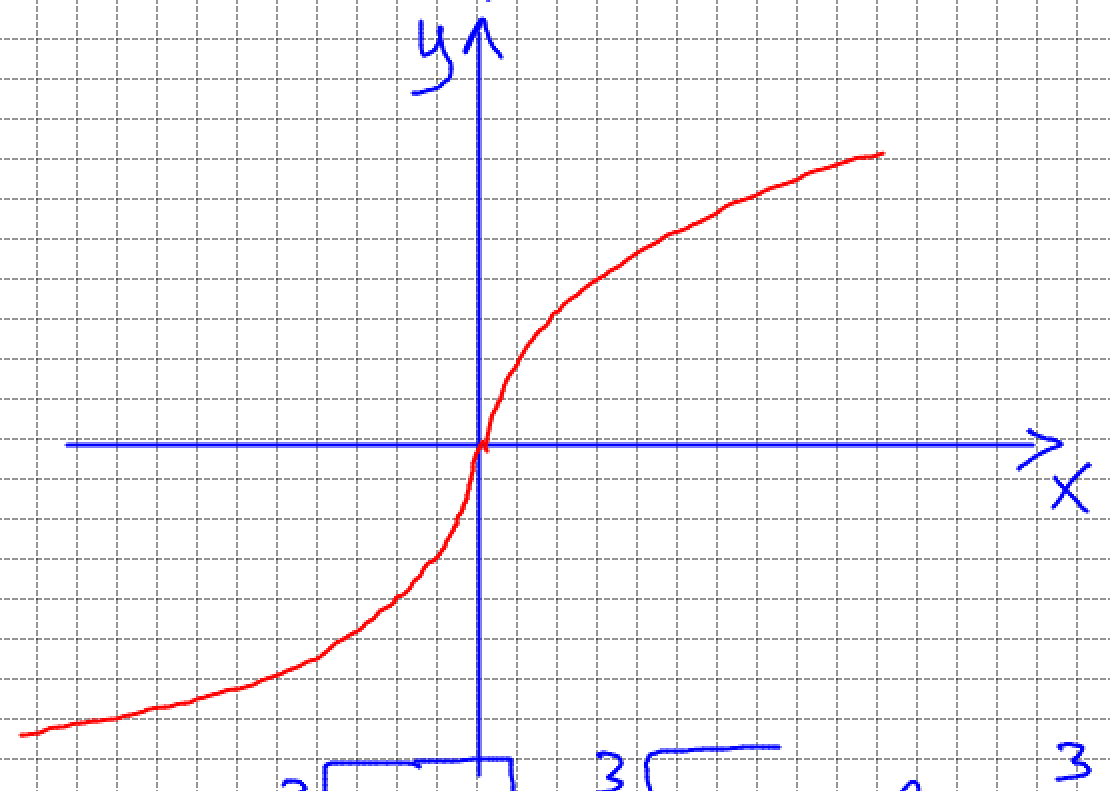
Se $$\lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h} = +\infty \quad (-\infty)$$

la funzione $y=f(x)$ non è derivabile in $x=x_0$ e $P(x_0; f(x_0))$ è un PUNTO DI FLESSO A TANGENTE VERTICALE.

ES

$$y=\sqrt[3]{x} \qquad x_0=0 \qquad D(f(x))=\frac{1}{3\sqrt[3]{x^2}}$$

$$\lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h} \overset{(\bullet)}{=} \lim_{h \to 0} \frac{\sqrt[3]{h}}{h} = +\infty$$



$$(\bullet) = \lim_{h \to 0} \frac{\sqrt[3]{0+h}-\sqrt[3]{0}}{h} = \lim_{h \to 0} \frac{\sqrt[3]{h}}{h} =$$

$$= \lim_{h \to 0} \sqrt[3]{\frac{h}{h^3}} = \lim_{h \to 0} \sqrt[3]{\frac{1}{h^2}} = \lim_{h \to 0} \frac{1}{\sqrt[3]{h^2}} =$$

$= +\infty$ Il punto $x_0=0$ per $f(x)=\sqrt[3]{x}$ è un punto di flesso a tangente verticale.

CUSPIDE

Se $\lim\limits_{h \to 0^-} \dfrac{f(x_0+h)-f(x_0)}{h} = -\infty$ e

$\lim\limits_{h \to 0^+} \dfrac{f(x_0+h)-f(x_0)}{h} = +\infty$

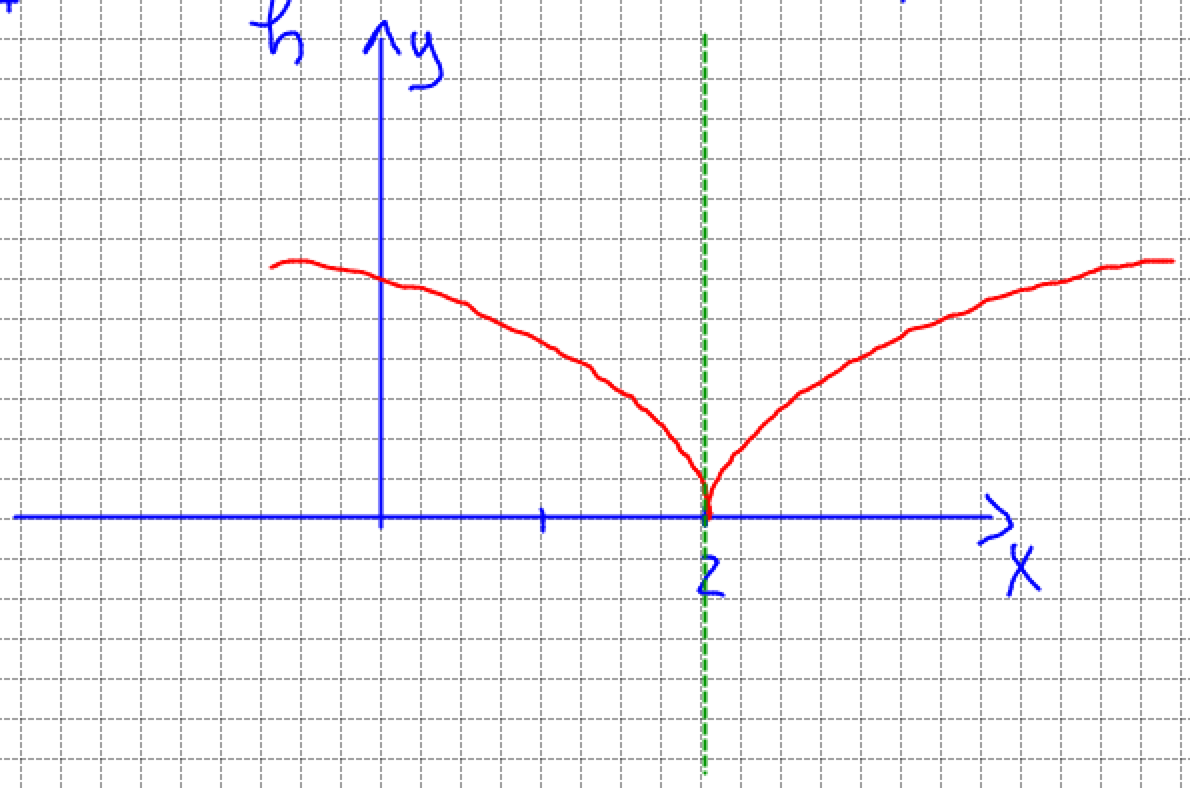
il punto $x = x_0 \in D(f(x))$ si chiama CUSPIDE

ES:

$y = \sqrt[3]{(x-2)^2}$ $\qquad x_0 = 2$

$$\lim_{h \to 0^-} \frac{f(2+h)-f(2)}{h} = -\infty \qquad (\cdot)$$

$$\lim_{h \to 0^+} \frac{f(2+h)-f(2)}{h} = +\infty \qquad (\cdot\cdot)$$



$$(\cdot) = \lim_{h \to 0^-} \frac{\sqrt[3]{(\cancel{2}+h-\cancel{2})^2} - \sqrt[3]{\cancel{(2-2)^2}}}{h} =$$

$$= \lim_{h \to 0^-} \frac{\sqrt[3]{h^2}}{h} = \lim_{h \to 0^-} \sqrt[3]{\frac{h^2}{h^3}} = \lim_{h \to 0^-} \sqrt[3]{\frac{1}{h}} = -\infty$$

$$(\cdot\cdot) \lim_{h \to 0^+} \frac{\sqrt[3]{(\cancel{2}+h-\cancel{2})^2} - \sqrt[3]{\cancel{(2-2)^2}}}{h} =$$

$$= \lim_{h \to 0^+} \frac{\sqrt[3]{h^2}}{h} = \lim_{h \to 0^+} \sqrt[3]{\frac{\cancel{h^2}}{h^3}} = \lim_{h \to 0^+} \sqrt[3]{\frac{1}{h}} = +\infty$$