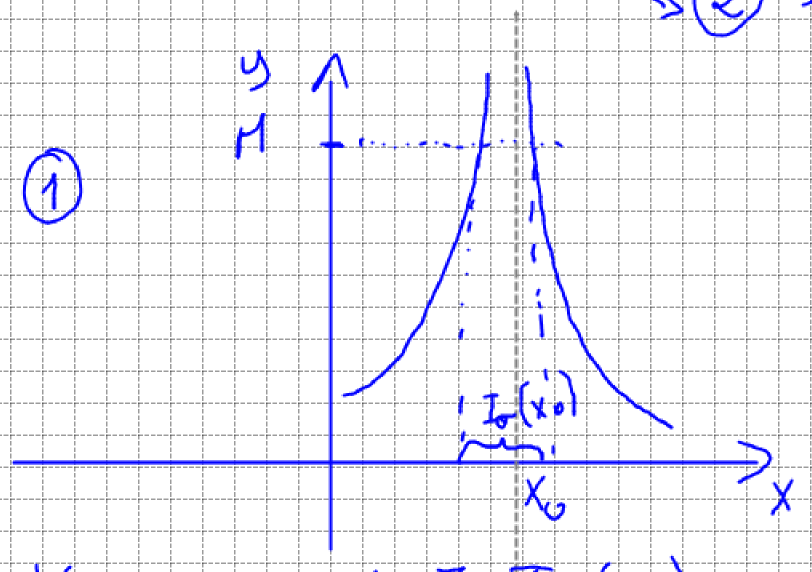
LIMITE INFINITO-FINITO

$\lim_{x\to x_0} f(x) = \infty$ → ① $\lim_{x\to x_0} f(x) = +\infty$

→ ② $\lim_{x\to x_0} f(x) = -\infty$

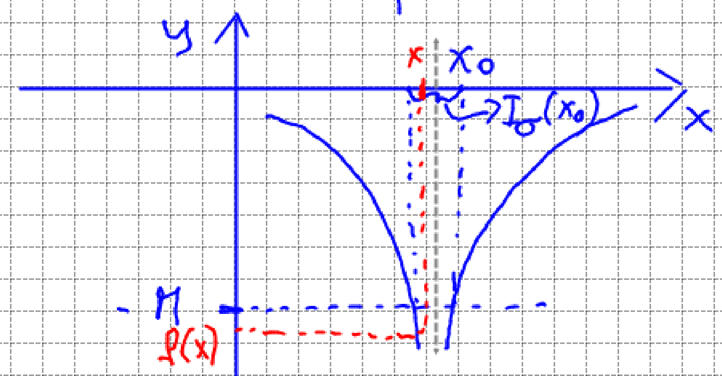
①



∀ M > 0 "grande" ∃ $I_M(+\infty)$ e corrispondentemente ∃ $I_\sigma(x_0)$ / ∀ x ∈ $I_\sigma(x_0)$ si ha che $f(x) \in I_M(+\infty)$ cioè:

$$f(x) > M$$

②



∀ -M < 0 "grande" ∃ $I_{-M}(-\infty)$ e corrispondentemente ∃ $I_\sigma(x_0)$ / ∀ x ∈ $I_\sigma(x_0)$ si ha che $f(x) \in I_{-M}(-\infty)$ cioè:

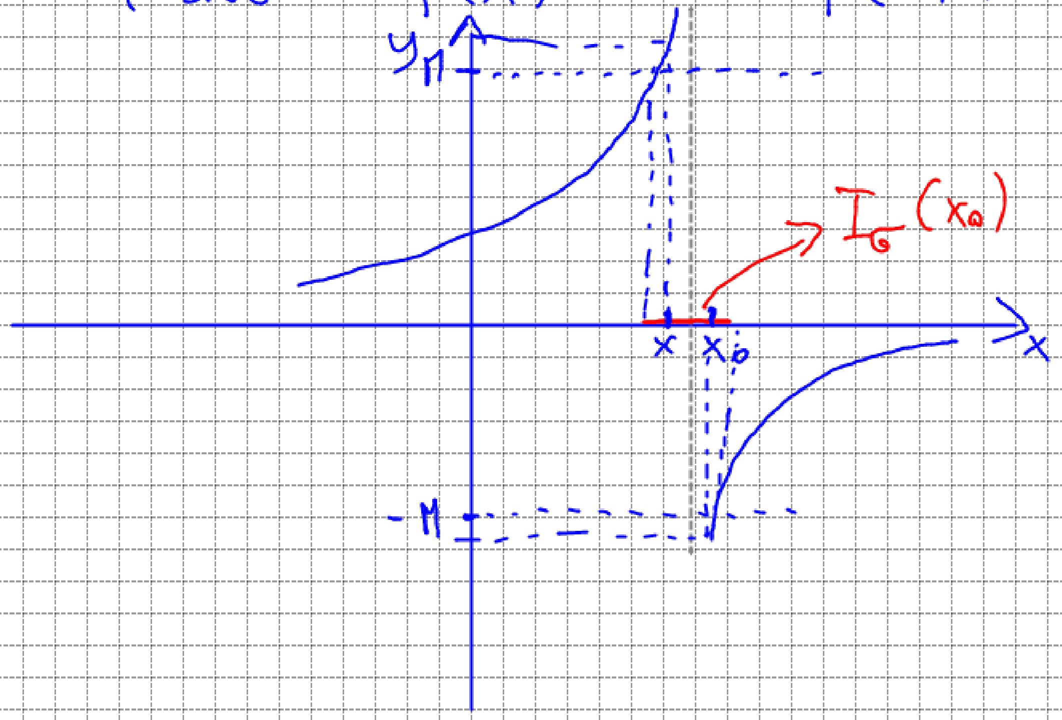
$$f(x) < -M$$

$$\boxed{\lim_{x\to x_0} f(x) = \infty}$$

∀ M > 0 ∃ $I(\infty) = I(-\infty) \cup I(+\infty)$ e corrispondentemente ∃ $I_\sigma(x_0)$ / ∀ x ∈ $I_\sigma(x_0)$ si ha $f(x) \in I(\infty)$ ovvero

$$|f(x)| > M$$

e cioè $f(x) < -M \cup f(x) > M$



LIMITE SINISTRO

$$\boxed{\lim_{x \to x_0^-} f(x) = +\infty}$$

$\forall M > 0 \; \exists \; I_M(+\infty)$ e corrisp. $\exists \; I_\sigma^-(x_0) \; / \; \forall x \in I_\sigma^-(x_0)$ si ha $f(x) > M$

$$\boxed{\lim_{x \to x_0^-} f(x) = -\infty}$$

$\forall -M < 0 \; \exists \; I_{-M}(-\infty)$ e corrisp. $\exists \; I_\sigma^-(x_0) \; / \; \forall x \in I_\sigma^-(x_0)$ si ha $f(x) < -M$

LIMITE DESTRO

$$\boxed{\lim_{x \to x_0^+} f(x) = +\infty}$$

$\forall M > 0 \; \exists \; I_M(+\infty)$ e corrisp. $\exists \; I_\sigma^+(x_0) \; / \; \forall x \in I_\sigma^+(x_0)$ si ha $f(x) > M$

$$\boxed{\lim_{x \to x_0^+} f(x) = -\infty}$$

$\forall -M < 0 \; \exists \; I_{-M}(-\infty)$ e corrisp. $\exists \; I_\sigma^+(x_0) \; / \; \forall x \in I_\sigma^+(x_0)$ si ha che $f(x) < -M$

ESEMPIO

Verificare il seguente limite

$$\lim_{x\to 0^+}\left[\frac{1}{x}+\frac{3}{x^2}\right]=+\infty$$

$\forall M>0\ \exists\ I_M(+\infty)$ e corrisp. $\exists\ I_\sigma^+(0)\ /\ \forall x\in I_\sigma^+(0)$

si ha $f(x)>M$ ovvero

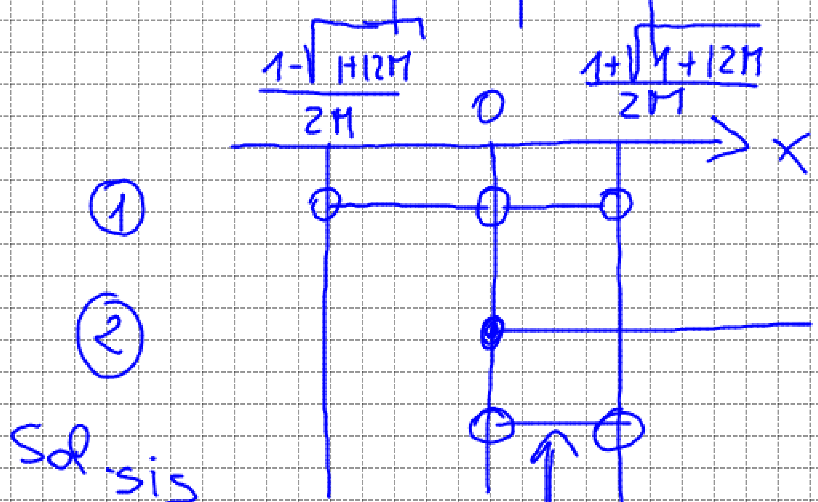
$$\begin{cases}\dfrac{1}{x}+\dfrac{3}{x^2}>M & ① \\ x\geq 0 & ②\end{cases}\qquad \begin{cases}\dfrac{x+3-Mx^2}{x^2}>0 \\ x\geq 0\end{cases}$$

① $N_1>0 \quad -Mx^2+x+3>0 \qquad x_{1,2}=\dfrac{-1\pm\sqrt{1+12M}}{-2M}=\dfrac{1\mp\sqrt{1+12M}}{2M}$

$$\frac{1-\sqrt{1+12M}}{2M}<x<\frac{1+\sqrt{12M+1}}{2M}$$

$D_1)>0 \quad x^2>0$ sempre + escluso $x=0$

		$\frac{1-\sqrt{1+12M}}{2M}$		0		$\frac{1+\sqrt{1+12M}}{2M}$	$\to x$
N_1	$-$	0	$+$		$+$	0	$-$
D_1	$+$		$+$	0	$+$		$+$
①	$-$	0	$+$	0	$+$	0	$-$



$$x\in\left(0\,;\,\frac{1+\sqrt{1+12M}}{2M}\right)=I_\sigma^+(0)$$