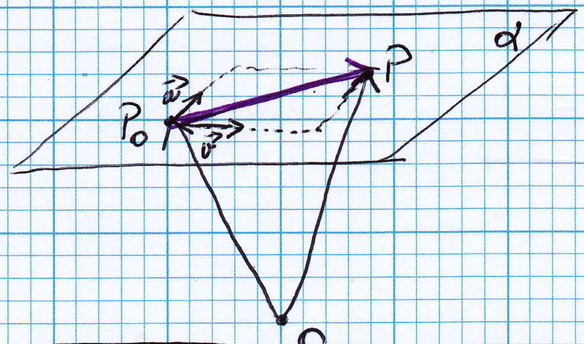
## EQUAZIONE DEL PIANO IN FORMA VETTORIALE




$$\overline{OP} = \overline{OP_0} + \overline{P_0P}$$

$$\boxed{\overrightarrow{OP} = \overrightarrow{OP_0} + \lambda \vec{w} + \mu \vec{v}}$$

equazione del piano in forma vettoriale

## EQUAZIONE DEL PIANO IN FORMA PARAMETRICA

$\overrightarrow{OP} = (x; y; z)$ $\quad \overrightarrow{OP_0} = (x_0; y_0; z_0)$ $\quad u(u_1 u_2 u_3)$ $\quad v(v_1 v_2 v_3)$

$$(x; y; z) = (x_0; y_0; z_0) + \lambda(u_1; u_2; u_3) + \mu(v_1; v_2; v_3)$$

$$\boxed{\begin{cases} x = x_0 + \lambda u_1 + \mu v_1 \\ y = y_0 + \lambda w_2 + \mu v_2 \\ z = z_0 + \lambda u_3 + \mu v_3 \end{cases}} \qquad \lambda, \mu \in \mathbb{R}$$

equazione del piano in forma parametrica.

## EQUAZIONE DEL PIANO IN FORMA CARTESIANA

$$\begin{vmatrix} x - x_0 & y - y_0 & z - z_0 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = 0$$

$$\boxed{ax + by + cz + d = 0} \qquad a, b, c, d \in \mathbb{R}$$

equazione del piano in forma cartesiana

ESEMPIO

Determinare l'equazione del piano per $P_0(1;4;-1)$ e parallelo ai vettori $\vec{u}=(-1;-2;-3)$ $\vec{v}(5;0;4)$

$$\overrightarrow{OP}=\overrightarrow{OP_0}+\lambda\vec{u}+\mu\vec{v}$$

$$(x;y;z)=(1;4;-1)+\lambda(-1;-2;-3)+\mu(5;0;4)$$

$$\begin{cases} x=1-\lambda+5\mu \\ y=4-2\lambda \\ z=-1-3\lambda+4\mu \end{cases} \qquad \begin{cases} x-1=-\lambda+5\mu \\ y-4=-2\lambda \\ z+1=-3\lambda+4\mu \end{cases}$$ eq. piano (forma parametrica)

$$\begin{vmatrix} x-1 & y-4 & z+1 \\ -1 & -2 & -3 \\ 5 & 0 & 4 \end{vmatrix}=0 \qquad (x-1)(-8)+(y-4)(-15+4)+(z+1)(10)=0$$

$$-8x+8-11y+44+10z+10=0$$

$$\boxed{8x+11y-10z-62=0}$$ eq. piano (forma cartesiana)

## EQUAZIONI PIANI PARTICOLARI

Se $P_0\equiv O$ si ha $O\in\alpha$ e

$$\overrightarrow{OP}=\lambda\vec{u}+\mu\vec{v} \qquad \vec{u}=(u_1;u_2;u_3) \quad \vec{v}=(v_1;v_2;v_3)$$

$$(x,y,z)=\lambda(u_1;u_2;u_3)+\mu(v_1;v_2;v_3)$$

$$\begin{vmatrix} x & y & z \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}=0 \qquad x(u_2v_3-u_3v_2)+y(u_2v_1-u_1v_2)+z(u_1v_2-u_2v_1)=0$$

ponendo $a=u_2v_3-u_3v_2$ $\quad b=u_2v_1-u_1v_2$ $\quad c=u_1v_2-u_2v_1$

si ha $\boxed{ax+by+cz=0}$

## PIANO xy

$$\begin{vmatrix} x & y & z \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 0 \qquad \boxed{z=0}$$

## PIANO xz

$$\begin{vmatrix} x & y & z \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0 \qquad \boxed{y=0}$$

## PIANO yz

$$\begin{vmatrix} x & y & z \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0 \qquad \boxed{x=0}$$

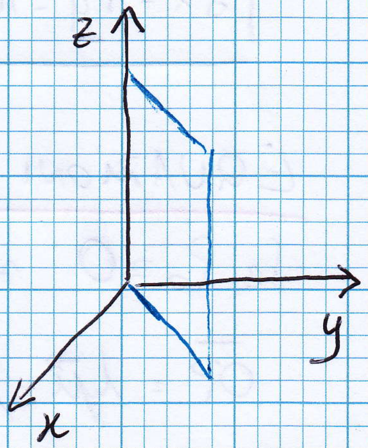
## PIANO PER L'ASSE z:

$$\begin{vmatrix} x & y & z \\ \lambda & \mu & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$\mu x - \lambda y = 0$

$\mu = a \quad -\lambda = b$

$\boxed{ax + by = 0}$



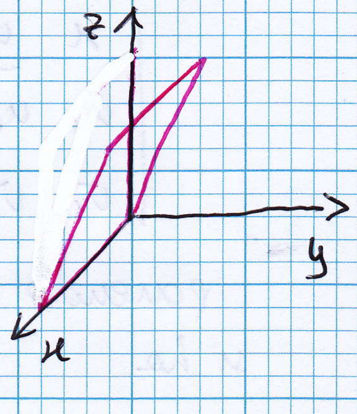

## PIANO PER L'ASSE x:

$$\begin{vmatrix} x & y & z \\ 0 & \lambda & \mu \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$-\mu y + \lambda z = 0$

$\lambda = c \quad -\mu = b$

$\boxed{by + cz = 0}$



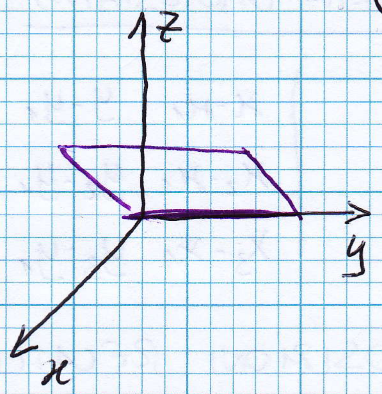
## PIANO PER L'ASSE Y

$$\begin{vmatrix} x & y & z \\ \lambda & 0 & \mu \\ 0 & 1 & 0 \end{vmatrix} = 0$$

$P(x; y; z)$

$P_0(\lambda; 0; \mu)$

$j = (0; 1; 0)$




## PIANO PARALLELO ALL'ASSE Z:

$$ax + by + d = 0$$

## PIANO PARALLELO ALL'ASSE X:

$$by + cz + d = 0$$

## PIANO PARALLELO ALL'ASSE Y:

$$ax + cz + d = 0$$

## PIANO PER TRE PUNTI

### Forma vettoriale

Dati A, B, C 3 punti non allineati e $\alpha$ il piano da essi determinato si ha

$$\boxed{\overline{OP} = \overline{OA} + \lambda \overline{AB} + \mu \overline{AC}} \qquad \lambda, \mu \in \mathbb{R}$$

### Forma parametrica:

$\overline{OA} = (x_1; y_1; z_1)$ $\quad \overline{OB} = (x_2; y_2; z_2)$ $\quad \overline{OC} = (x_3; y_3; z_3)$

$\overline{AB} = (x_2 - x_1; y_2 - y_1; z_2 - z_1)$ $\quad \overline{AC} = (x_3 - x_1; y_3 - y_1; z_3 - z_1)$ $\quad P(x; y; z)$

quindi:

$$\begin{cases} x = x_1 + \lambda (x_2 - x_1) + \mu (x_3 - x_1) \\ y = y_1 + \lambda (y_2 - y_1) + \mu (y_3 - y_1) \\ z = z_1 + \lambda (z_2 - z_1) + \mu (z_3 - z_1) \end{cases}$$