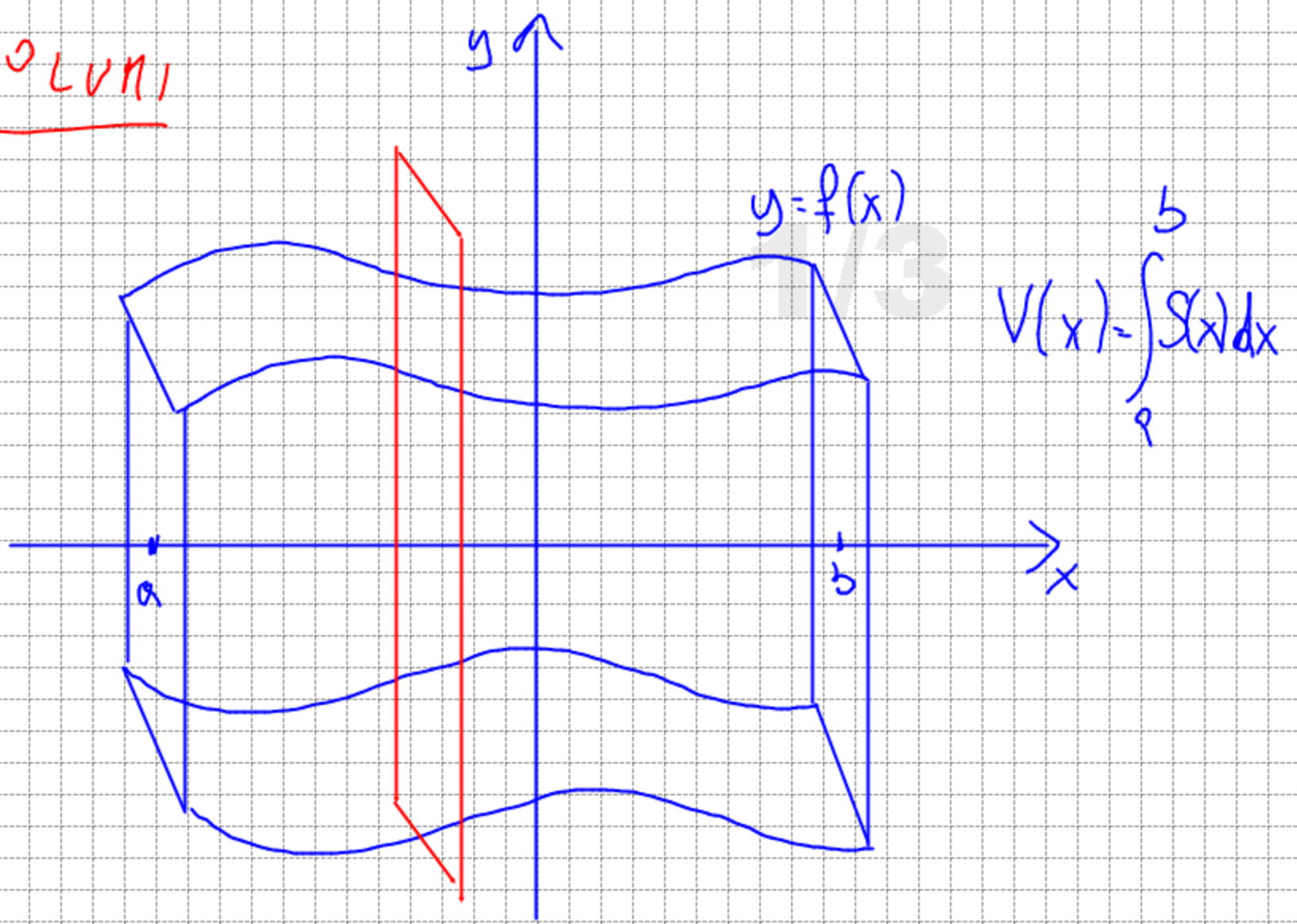
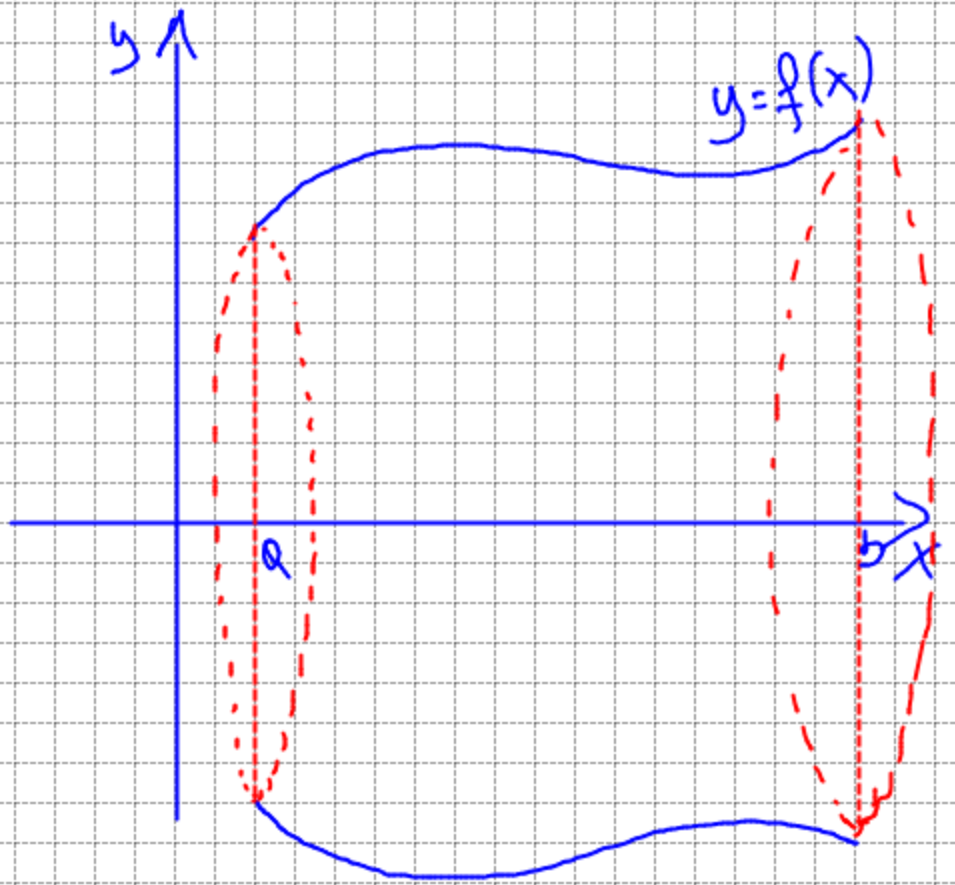


## VOLUMI

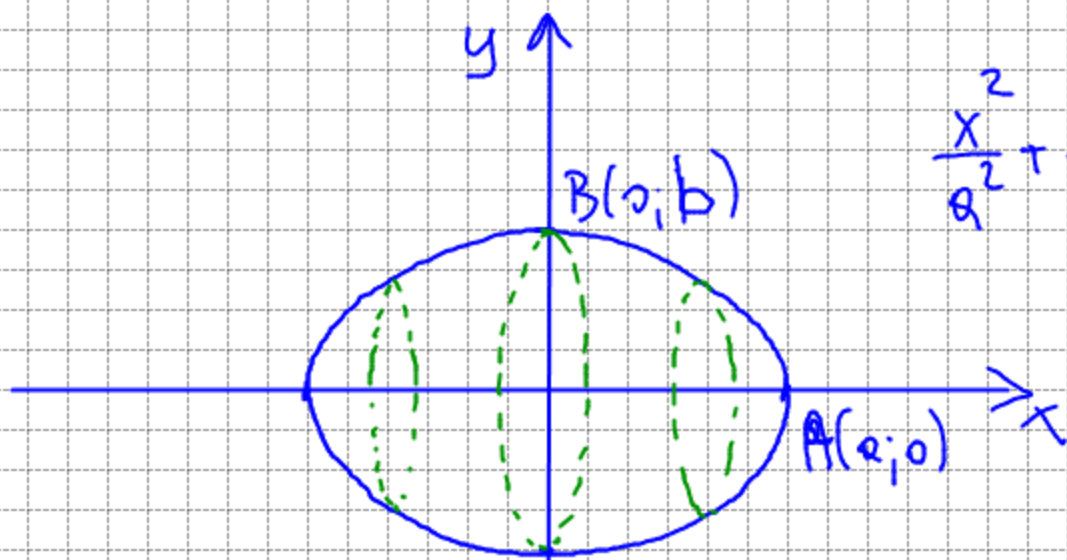


## VOLUME DEI SOLIDI DI ROTAZIONE

$$V = \pi \int_a^b f^2(x) dx$$



## VOLUME ELLISSOIDE



$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} \quad y^2 = b^2 \left( 1 - \frac{x^2}{a^2} \right) \quad y = \pm b \sqrt{1 - \frac{x^2}{a^2}}$$

$$y = b \sqrt{1 - \frac{x^2}{a^2}} \quad f(x)$$

$$V = \pi \int_{-a}^a f^2(x) dx = 2\pi \int_0^a f^2(x) dx =$$

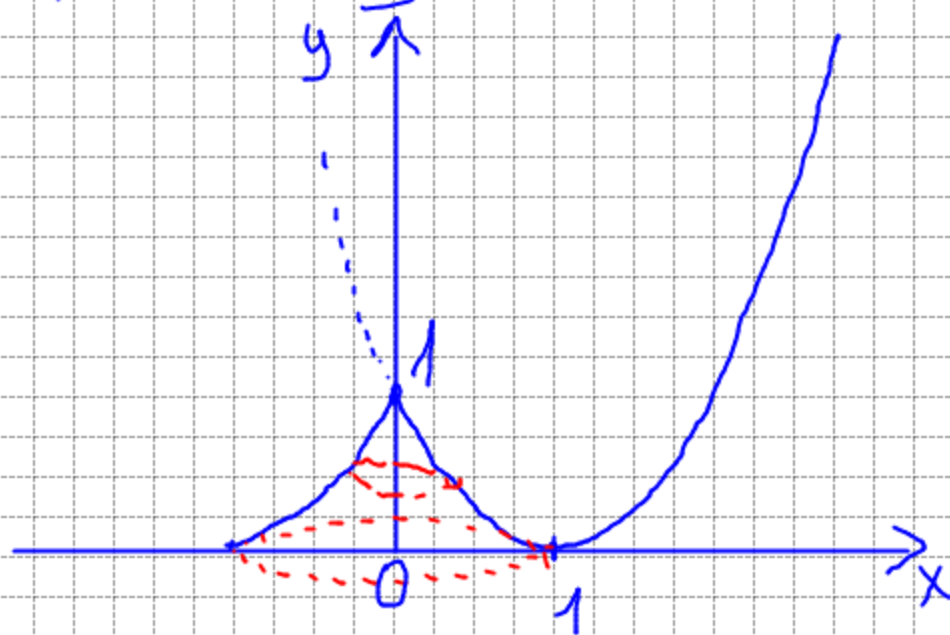
$$= 2\pi b^2 \int_0^a \sqrt{1 - \frac{x^2}{a^2}} dx = \frac{2\pi b^2}{a^2} \int_0^a (a^2 - x^2) dx =$$

$$= \frac{2\pi b^2}{a^2} \left[ a^2 x - \frac{x^3}{3} \right]_0^a =$$

$$= \frac{2\pi b^2}{a^2} \left[ a^3 - \frac{a^3}{3} \right] = \frac{2\pi b^2}{a^2} \frac{2a^3}{3} = \frac{4\pi a b^2}{3}$$

## ESERCIZIO

Data la parabola  $y = x^2 - 2x + 1$



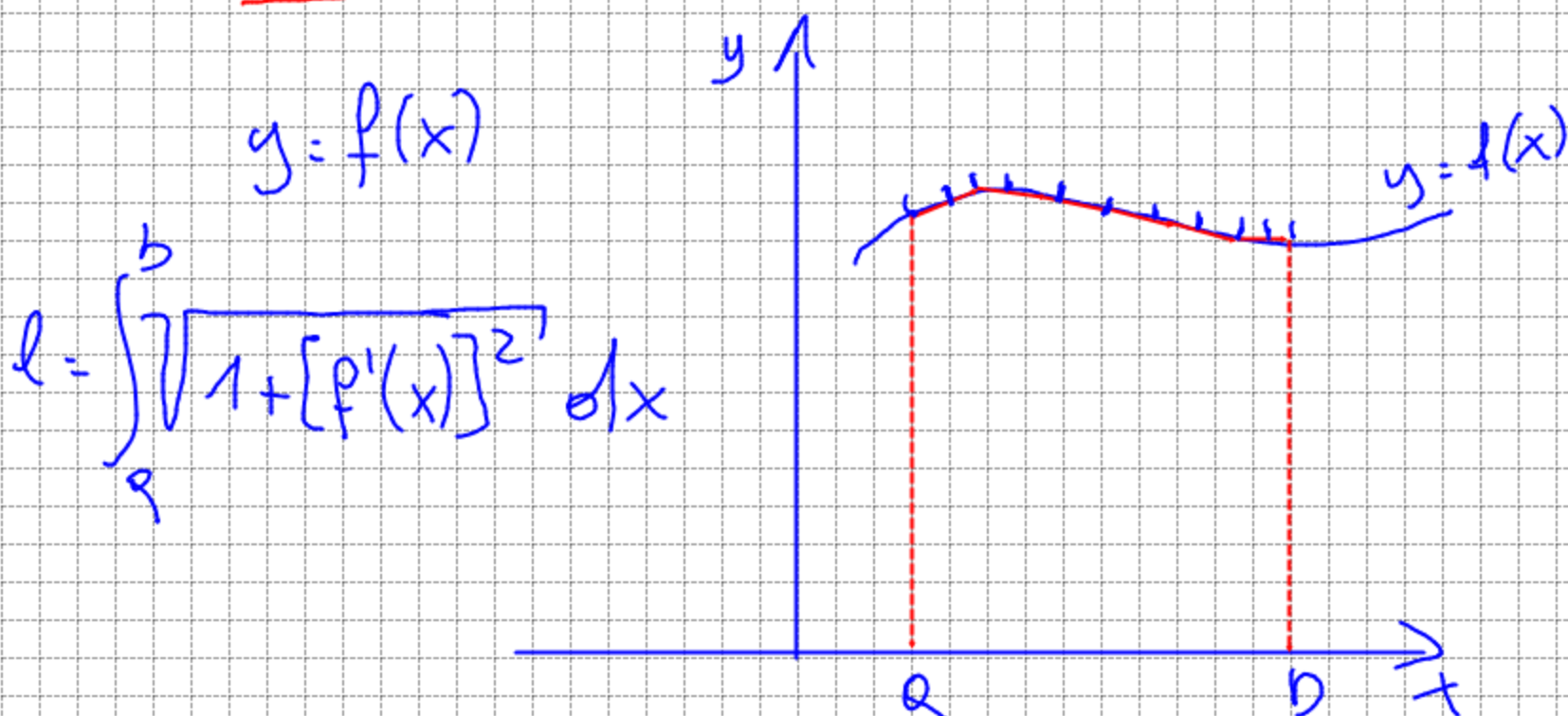
$x = 1 \pm \sqrt{y}$  Tra 0 e 1 la curva  $1 - \sqrt{y} = x$

$$V = \pi \int_0^1 [1 - \sqrt{y}]^2 dy = \pi \int_0^1 (1 + y - 2\sqrt{y}) dy =$$

$$= \pi \left[ y + \frac{y^2}{2} - 2 \frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_0^1 =$$

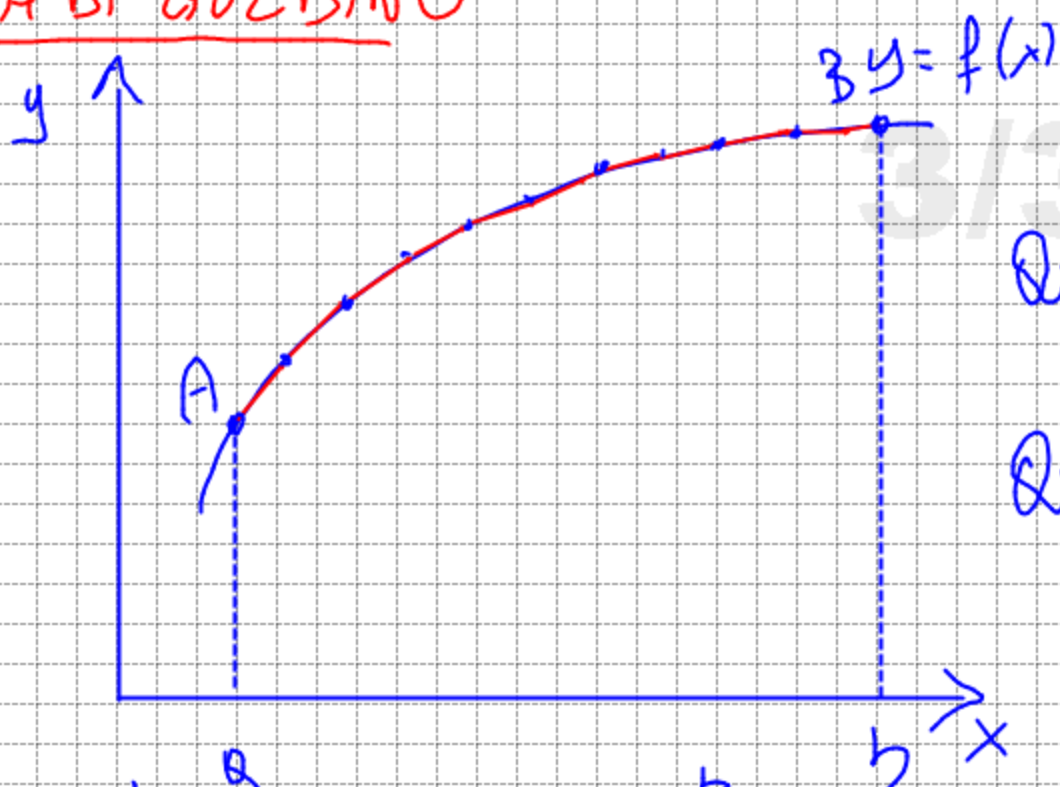
$$= \pi \left( 1 + \frac{1}{2} - \frac{4}{3} \right) = \frac{\pi}{6}$$

## LUNGHEZZA ARCO DI CURVA



$$l = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

## TEOREMA DI GULDINO



$$Area_y = 2\pi |x_G| L$$

$$Area_x = 2\pi |y_G| L$$

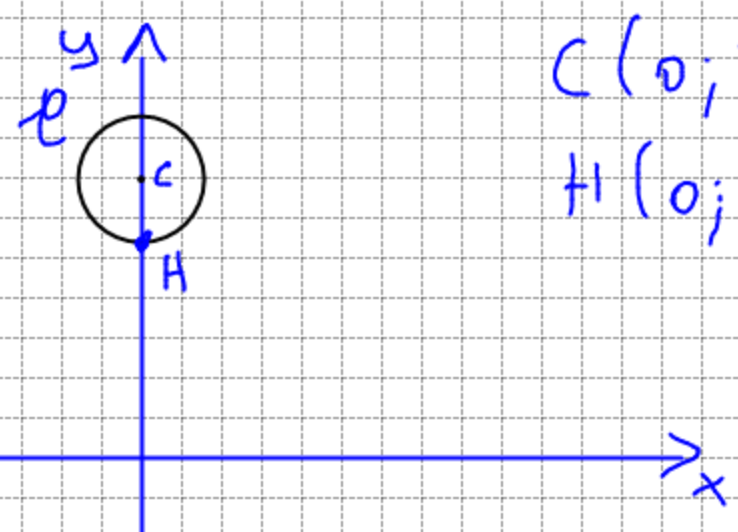
$$m \int_a^b \sqrt{1+f'(x)^2} dx \leq \int_a^b f(x) \sqrt{1+f'(x)^2} dx \leq M \int_a^b \sqrt{1+f'(x)^2} dx$$

$$\text{con } \int_a^b \sqrt{1+f'(x)^2} dx = L$$

$$m \leq \frac{\int_a^b f(x) \sqrt{1+f'(x)^2} dx}{L} \leq M$$

$y_G$

## AREA DEL TORO



$$C(0; R)$$

$$H(0; R-\pi)$$

$$\overline{CH} = \pi$$

$$C: (x-0)^2 + (y-R)^2 = R^2 \Leftrightarrow x^2 + (y-R)^2 = R^2$$

$$x^2 + y^2 + R^2 - 2Ry = R^2$$

$$y_G = R$$

lunghezza  
circonferenza

$$Q_T = 2\pi |y_G| L = 2\pi R \cdot 2\pi R = 4\pi^2 R^2$$