

FORMULA DI DE MOIVRE

Determiniamo le potenze intere dei numeri complessi:

$$z = \rho (\cos \theta + i \operatorname{sen} \theta) \quad z^n = ? \quad n \in \mathbb{N}$$

Dimostriamo $z^n = \rho^n (\cos n\theta + i \operatorname{sen} n\theta) \quad (*)$

per induzione:

1) la formula $(*)$ è vera per $n=1$:

$$z^1 = \rho^1 (\cos 1\theta + i \operatorname{sen} 1\theta)$$

2) se la formula $(*)$ è vera per n cioè:

$$z^n = \rho^n (\cos n\theta + i \operatorname{sen} n\theta)$$

3) dimostriamo $(*)$ per $n+1$:

$$\begin{aligned} z^{n+1} &= z^n \cdot z = \rho^n (\cos n\theta + i \operatorname{sen} n\theta) \rho (\cos \theta + i \operatorname{sen} \theta) = \\ &= \rho^{n+1} \left[\begin{array}{l} \cos n\theta \cos \theta - \operatorname{sen} n\theta \operatorname{sen} \theta \\ + i (\cos n\theta \operatorname{sen} \theta + \operatorname{sen} n\theta \cos \theta) \end{array} \right] = \\ &= \rho^{n+1} \left[\cos (n\theta + \theta) + i \operatorname{sen} (n\theta + \theta) \right] = \\ &= \rho^{n+1} \left[\cos (n+1)\theta + i \operatorname{sen} (n+1)\theta \right] \quad \text{C.V.D.} \end{aligned}$$

ESEMPIO

$$(1 - \sqrt{3}i)^3$$

$$a=1 \quad b=-\sqrt{3}$$

$$\rho = \sqrt{1+3} = 2$$

$$\begin{cases} \cos \theta = \frac{1}{2} \\ \operatorname{sen} \theta = -\frac{\sqrt{3}}{2} \end{cases} \quad \theta = -\frac{\pi}{3}$$

$$\begin{aligned} (1 - i\sqrt{3})^3 &= \left[2 \left(\cos\left(-\frac{\pi}{3}\right) + i \operatorname{sen}\left(-\frac{\pi}{3}\right) \right) \right]^3 = 2^3 \left[\cos(-\pi) + i \operatorname{sen}(-\pi) \right] = \\ &= 8(-1) = -8 \end{aligned}$$

RADICE m-ESIMA DI UN NUMERO COMPLESSO

Dato un numero complesso $z = \rho(\cos \theta + i \sin \theta)$

si chiama RADICE m-ESIMA di z quel numero complesso w

Tale che

$$\underline{w^m = z}$$

Sia $w = \rho_1(\cos \theta_1 + i \sin \theta_1)$ allora

$$w^m = \rho_1^m (\cos m\theta_1 + i \sin m\theta_1) \quad (\text{DE MOIVRE})$$

Si come $w^m = z \Leftrightarrow \rho_1^m (\cos m\theta_1 + i \sin m\theta_1) = \rho (\cos \theta + i \sin \theta)$

$$\Leftrightarrow \rho_1^m = \rho \quad \text{e} \quad m\theta_1 = \theta + 2k\pi$$

$$\Leftrightarrow \boxed{\begin{aligned} \rho_1 &= \sqrt[m]{\rho} \\ \theta_1 &= \frac{\theta}{m} + \frac{2k\pi}{m} \end{aligned}}$$

$$\sqrt[m]{\rho (\cos \theta + i \sin \theta)} = \sqrt[m]{\rho} \left(\cos \left(\frac{\theta + 2k\pi}{m} \right) + i \sin \left(\frac{\theta + 2k\pi}{m} \right) \right)$$

ESEMPIO

Calcolare le radici cubiche di 1:

$$\begin{aligned} \rho &= 1 \\ b &= 0 \quad \rho = 1 \\ \cos \theta &= 1 \quad \theta = 0 + 2k\pi \\ \sin \theta &= 0 \end{aligned}$$

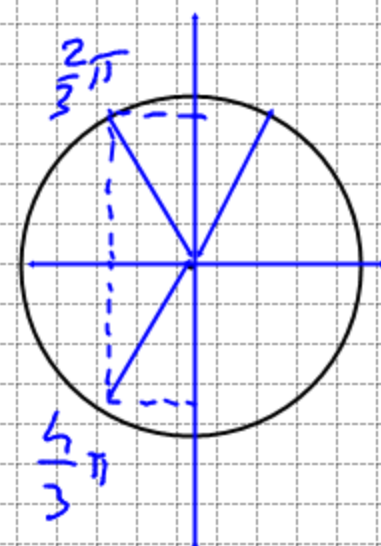
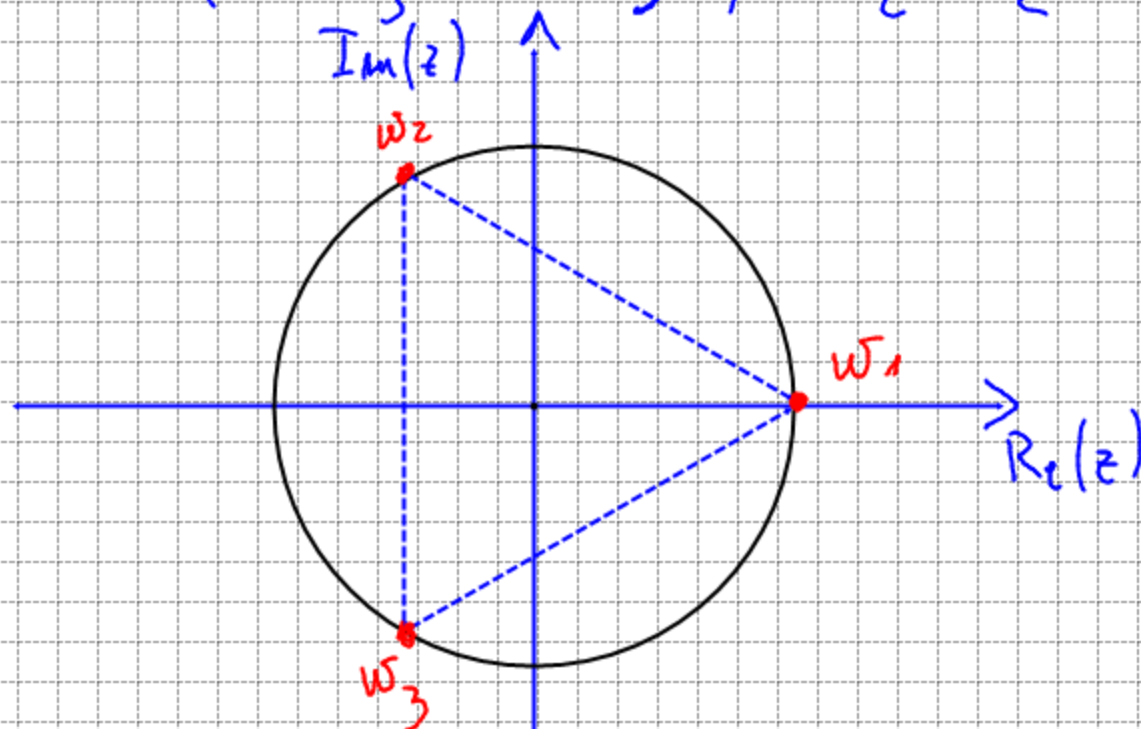
$$1 = 1 (\cos 0 + i \sin 0)$$

$$\sqrt[3]{1} = \sqrt[3]{1} \left(\cos \frac{0 + 2k\pi}{3} + i \sin \frac{0 + 2k\pi}{3} \right) \quad k = 0, 1, 2$$

$$w_1 = (\cos 0 + i \sin 0) = 1$$

$$w_2 = \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right) = -\frac{1}{2} + i \frac{\sqrt{3}}{2}$$

$$w_3 = \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right) = -\frac{1}{2} - i \frac{\sqrt{3}}{2}$$



$$\sqrt[4]{1} = \sqrt[4]{1} (\cos \theta + i \sin \theta) = 1 \left[\cos \left(\frac{0 + 2k\pi}{4} \right) + i \sin \left(\frac{0 + 2k\pi}{4} \right) \right]$$

$$k=0$$

$$w_1 = 1$$

$$k=1$$

$$w_2 = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} = i$$

$$k=2$$

$$w_3 = \cos \pi + i \sin \pi = -1$$

$$k=3$$

$$w_4 = \cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} = -i$$

