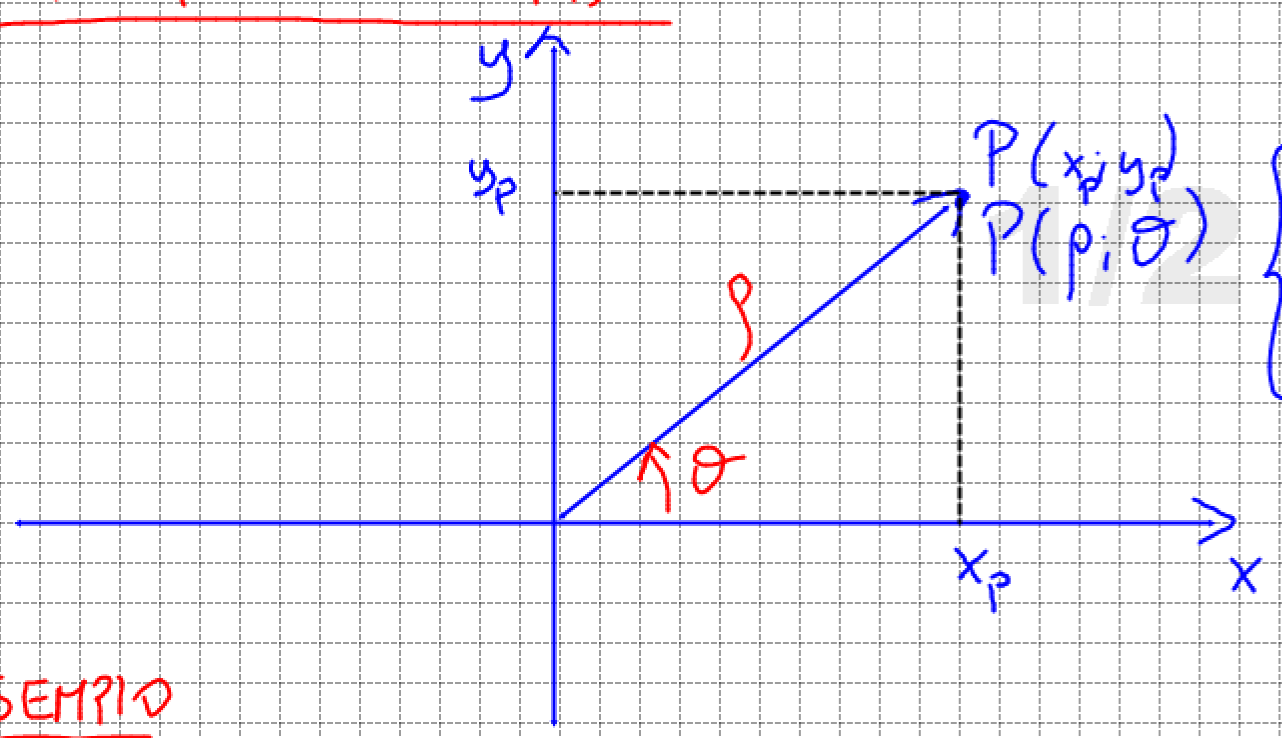
COORDINATE POLARI NEL PIANO



$$\begin{cases} x_P = \rho \cos\theta \\ y_P = \rho \operatorname{sen}\theta \end{cases}$$

ESEMPIO

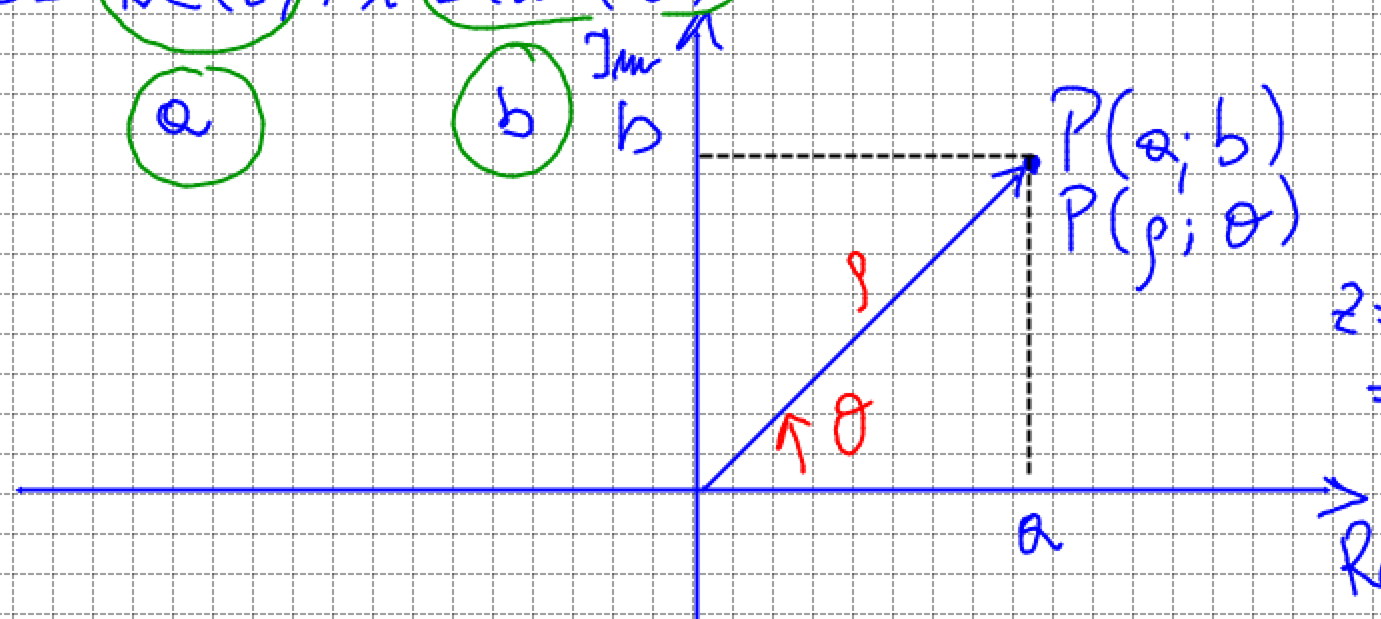
Quali sono le coordinate polari di $A(1; \sqrt{3})$?

$$\begin{cases} 1 = \rho\cos\theta \\ \sqrt{3} = \rho \operatorname{sen}\theta \end{cases} \qquad \frac{\rho \operatorname{sen}\theta}{\rho\cos\theta} = \frac{\sqrt{3}}{1} \Rightarrow \operatorname{tg}\theta = \sqrt{3}$$

$$\theta = \frac{\pi}{3} \qquad \rho^2\cos^2\theta + \rho^2 \operatorname{sen}^2\theta = 4 \qquad \rho^2 = 4 \qquad \rho = 2$$

$A\left(2; \frac{\pi}{3}\right)$ coordinate polari.

$$z = \underset{a}{Re(z)} + i\, \underset{b}{Im(z)}$$



$P(7; 45°)$

$$z = 7(\cos 45° + i \operatorname{sen} 45°) = 7\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$$

$$z = a + ib = \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}} + i\frac{b}{\sqrt{a^2+b^2}}\right)$$

$$\rho = \sqrt{a^2+b^2} \qquad \operatorname{sen}\theta = \frac{b}{\sqrt{a^2+b^2}} \qquad \cos\theta = \frac{a}{\sqrt{a^2+b^2}}$$

$$\boxed{z = \rho(\cos\theta + i \operatorname{sen}\theta)}$$

RAPPRESENTAZIONE TRIGONOMETRICA O POLARE DI Z

PRODOTTO (in forma polare)

$$z_1 = \rho_1(\cos\theta_1 + i \operatorname{sen}\theta_1) \qquad z_2 = \rho_2(\cos\theta_2 + i\operatorname{sen}\theta_2)$$

$$z_1 \cdot z_2 = \rho_1\rho_2\left[\cos\theta_1\cos\theta_2 + i\cos\theta_1 \operatorname{sen}\theta_2 + i \operatorname{sen}\theta_1\cos\theta_2 - \operatorname{sen}\theta_1 \operatorname{sen}\theta_2\right]$$

$$= \rho_1\rho_2\left[\cos(\theta_1+\theta_2) + i\operatorname{sen}(\theta_1+\theta_2)\right]$$

QUOZIENTE (in coordinate polari)

$$z_1 = \rho_1(\cos\theta_1 + i\operatorname{sen}\theta_1) \qquad z_2 = \rho_2(\cos\theta_2 + i\operatorname{sen}\theta_2) \neq 0.$$

$$\frac{z_1}{z_2} = \frac{\rho_1}{\rho_2}\frac{\cos\theta_1 + i\operatorname{sen}\theta_1}{\cos\theta_2 + i\operatorname{sen}\theta_2} = \frac{\rho_1}{\rho_2}\frac{(\cos\theta_1 + i\operatorname{sen}\theta_1)(\cos\theta_2 - i\operatorname{sen}\theta_2)}{(\cos\theta_2 + i\operatorname{sen}\theta_2)(\cos\theta_2 - i\operatorname{sen}\theta_2)}$$

$$\frac{z_1}{z_2} = \frac{\rho_1}{\rho_2}\,\frac{(\cos\theta_1\cos\theta_2 - i\cos\theta_1\,\mathrm{sen}\,\theta_2 + i\,\mathrm{sen}\,\theta_1\cos\theta_2 + \mathrm{sen}\,\theta_1\,\mathrm{sen}\,\theta_2)}{\cos^2\theta_2 + \mathrm{sen}^2\theta_2} =$$

$$\boxed{\frac{z_1}{z_2} = \frac{\rho_1}{\rho_2}\left[\frac{\cos(\theta_1-\theta_2) + i\,\mathrm{sen}(\theta_1-\theta_2)}{1}\right]}$$