$\boxed{(3)}$ $\Delta < 0$ $\displaystyle\int \frac{1}{ax^2+bx+c}\,dx$ $\Delta < 0$.

ESEMPIO

$x^2+x+1 = (x+m)^2+n$

$$x^2+x+1 = x^2+2\cdot\frac{1}{2}\cdot x+1+\frac{1}{4}-\frac{1}{4} = \left(x^2+2\,\frac{1}{2}\,x+\frac{1}{4}\right)+\frac{3}{4} = \left(x+\frac{1}{2}\right)^2+\frac{3}{4}$$

$$\int \frac{1}{x^2+x+1}\,dx = \int \frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}\,dx = \int \frac{1}{\frac{3}{4}\left[\frac{\left(x+\frac{1}{2}\right)^2}{\frac{3}{4}}+1\right]}\,dx =$$

$$= \frac{4}{3}\int \frac{1}{\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)^2+1}\,dx = \frac{4}{3}\int \frac{1}{\left[\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)\right]^2+1}\,dx =$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{4}{3}\int \frac{1}{\left[\underbrace{\boxed{\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)}}_{X}\right]^2+1}\,d\left(\overbrace{\boxed{\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)}}^{X}\right) =$$

$$= \frac{2\sqrt{3}}{3}\int \frac{1}{X^2+1}\,dX = \frac{2\sqrt{3}}{3}\,\operatorname{arctg}\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)+c.$$

ESEMPIO

$$\int \frac{3x+1}{x^2+9}\,dx = \int \frac{3x}{x^2+9}\,dx+\int \frac{1}{x^2+9}\,dx =$$

$$= \frac{3}{2}\int \frac{\overbrace{2x}^{f'(x)}}{\underbrace{x^2+9}_{f(x)}}\,dx+\int \frac{1}{x^2+9}\,dx =$$

$$= \frac{3}{2}\ln\left|x^2+9\right|+c_1+\frac{1\cdot 3}{9}\int \frac{1}{\left(\frac{x}{3}\right)^2+1}\,d\left(\frac{x}{3}\right) =$$

$$= \frac{3}{2}\ln\left|x^2+9\right|+c_1+\frac{1}{3}\operatorname{arctg}\left(\frac{x}{3}\right)+c_2$$

INTEGRALI CON MODULI

Data la funzione $y = f(x)$

- $\int f(x)\,dx$ rappresenta la famiglia delle primitive di $f(x)$
- $\int |f(x)|\,dx$ rappresenta la famiglia delle primitive di $|f(x)|$

$$|f(x)| \begin{cases} = f(x) & \text{se } f(x) \geq 0 \\ = -f(x) & \text{se } f(x) < 0 \end{cases}$$

ESEMPIO

$$\int |x|\,dx = \qquad |x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$

se $x \geq 0$

$$\int |x|\,dx = \int x\,dx = \frac{1}{2}x^2 + c$$

se $x < 0$

$$\int |x|\,dx = \int -x\,dx = -\frac{1}{2}x^2 + c$$

$$\int |x|\,dx = c + \begin{cases} \frac{1}{2}x^2 & \text{se } x \geq 0 \\ -\frac{1}{2}x^2 & \text{se } x < 0 \end{cases}$$

$$\int |x|\,dx = \frac{1}{2}x|x| + c$$