$$\int \frac{1}{2x+1}\,dx = \frac{1}{2}\int \frac{1}{2x+1}\,d(2x+1) = \frac{1}{2}\ln|2x+1| + c$$

$$\int \cos 2x\,dx = \frac{1}{2}\int \cos 2x\,d(2x) = \frac{1}{2}\,\mathrm{sen}\,2x + C$$

$$\int \frac{1}{\mathrm{sen}^2(3-x)}\,dx = \int -\frac{1}{\mathrm{sen}^2(3-x)}\,d(3-x) = \mathrm{ctg}(3-x) + C$$

INTEGRAZIONE DELLE FUNZIONI RAZIONALI

$$f(x)=\frac{N(x)}{D(x)} \qquad \begin{array}{r|l} N(x) & D(x) \\ \hline & Q(x) \\ R(x) & \end{array}$$

$$f(x)=\frac{N(x)}{D(x)} \qquad f(x)=Q(x)+\frac{R(x)}{D(x)}$$

$$\int f(x)\,dx=\int\frac{N(x)}{D(x)}\,dx=\int Q(x)\,dx+\int\frac{R(x)}{D(x)}\,dx$$

1° caso: $\displaystyle\int\frac{K}{ax+b}\,dx$ — $N(x)$ ha grado 0, $D(x)$ ha grado 1

$$=\frac{K}{a}\int\frac{1}{ax+b}\,d(ax+b)=\frac{K}{a}\ln|ax+b|+c$$

ESEMPIO

$$\int\frac{1-x^2+3x^4}{2-x}\,dx=$$

$$\begin{array}{l|l}
N(x)\ \boxed{3x^4+0x^3-x^2+0x+1} & \boxed{-x+2}\ D(x) \\
-3x^4+6x^3 & \boxed{-3x^3-6x^2-11x-22} \\
\hline
//\quad 6x^3-x^2 & \quad Q(x) \\
\quad -6x^3+12x^2 & \\
\hline
\quad //\quad 11x^2+0x & \\
\qquad -11x^2+22x & \\
\hline
\qquad //\quad 22x+1 & \\
\qquad\quad -22x+44 & \\
\hline
\qquad\quad //\quad 45 = R(x)=R &
\end{array}$$

$$\int\frac{3x^4-x^2+1}{-x+2}\,dx=\int(-3x^3-6x^2-11x-22)\,dx+\int\frac{45}{2-x}\,dx$$

$$=-\frac{3}{4}x^4-2x^3-\frac{11}{2}x^2-22x-45\ln|2-x|+c.$$

2° CASO:

$$\int \frac{gx+e}{ax^2+bx+c}\,dx \begin{cases} \boxed{(1)} & \Delta>0 \\ \boxed{(2)} & \Delta=0 \\ \boxed{(3)} & \end{cases}$$

$\boxed{(1)}$ $\Delta>0$

ESEMPIO

$$\int \frac{2x-1}{x^2-5x+6}\,dx = (\bullet) \qquad x^2-5x+6=(x-3)(x-2)$$

$$\frac{2x-1}{x^2-5x+6} = \frac{A}{(x-3)} + \frac{B}{(x-2)}$$

$$\frac{2x-1}{(x-3)(x-2)} = \frac{\overset{(A+B)x+(-2A-3B)}{Ax-2A+Bx-3B}}{(x-3)(x-2)} \qquad \begin{cases} A+B=2 \\ -2A-3B=-1 \end{cases}$$

$$\begin{cases} A=2-B \\ 2(2-B)+3B=1 \end{cases} \quad \begin{cases} A=2-B \\ 4+B=1 \end{cases} \quad \begin{cases} A=5 \\ B=-3 \end{cases}$$

$$\bullet = \int \frac{5}{(x-3)}\,dx - \int \frac{3}{(x-2)}\,dx = 5\ln|x-3| - 3\ln|x-2| + c$$

$\boxed{(2)}$ $\Delta=0 \qquad ax^2+bx+c=a(x-\alpha)^2$

ESEMPIO

$$\int \frac{x+1}{x^2-10x+25}\,dx = (\bullet\bullet) \qquad D(x^2-10x+25)=2x-10$$

$$(\bullet\bullet) = \frac{1}{2}\int \frac{2x\overset{=-10}{\overbrace{+2-12}}+12}{x^2-10x+25}\,dx = \frac{1}{2}\int \frac{2x-10}{x^2-10x+25}\,dx +$$

$$+ \int \frac{12}{x^2-10x+25}\,dx = \frac{1}{2}\Big[\ln|x^2-10x+25| +$$

$$+ 12\int \frac{1}{(x-5)^2}\,dx\Big] = \frac{1}{2}\ln|x^2-10x+25| +$$

$$+ \frac{12}{2}\,\frac{(-1)}{x-5} + c.$$