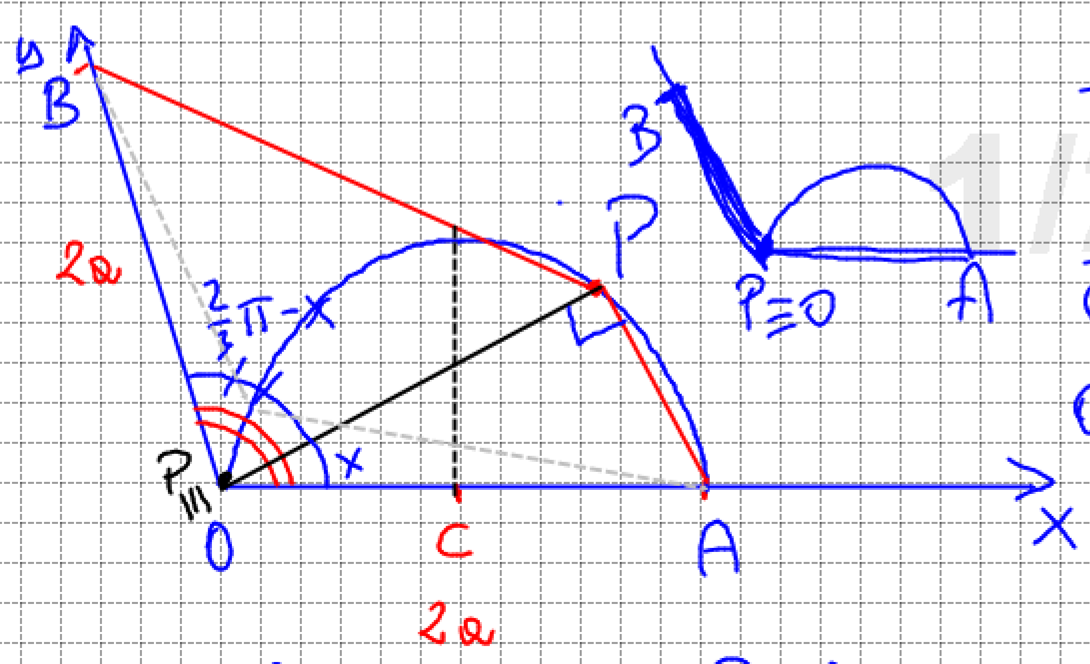
PROBLEMA



DATI

$\widehat{XOY} = \frac{2}{3}\pi$

$\overline{OA} = \overline{OB} = 2a$

OAPB è un quadrilatero convesso.

Determinare la posizione di P affinchè sia MASSIMA l'area del quadrilatero APBO al variare di P

Chiamamo $\widehat{AOP} = x$ $\qquad 0 \le x < \frac{\pi}{2}$ non abbiamo più un quadrilatero convesso. (*)

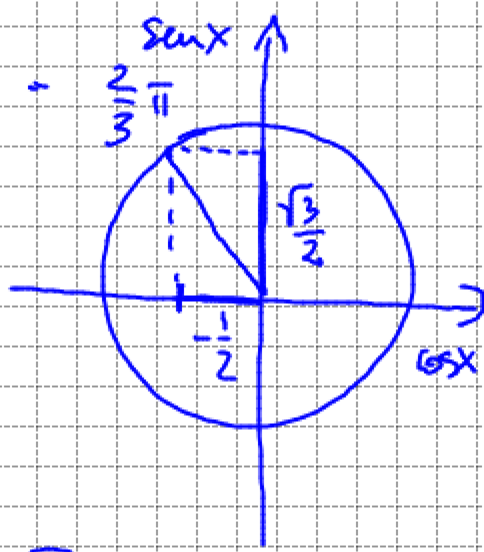
$$Q_{\triangle OAP}(x) = \frac{\overline{OP} \times \overline{PA}}{2} = \frac{2a\cos x \,(2a \operatorname{sen} x)}{2} = 2a^2 \operatorname{sen} x \cos x$$

$$Q_{OPB}(x) = \frac{1}{2}\,\overline{OP}\cdot\overline{OB}\,\operatorname{sen}\left(\frac{2}{3}\pi - x\right) = \frac{1}{2}\, 2a\cos x \cdot 2a \operatorname{sen}\left(\frac{2}{3}\pi - x\right) =$$

$$= 2a^2\cos x\left[\operatorname{sen}\frac{2}{3}\pi \cos x - \cos\frac{2}{3}\pi \operatorname{sen} x\right] =$$

$$= 2a^2\cos x\left[\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\operatorname{sen} x\right] =$$

$$= \sqrt{3}a^2\cos^2 x + a^2 \operatorname{sen} x\cos x$$



$$Q(x) = Q_{\widehat{OAP}}(x) + Q_{\widehat{OPB}}(x) = 3a^2 \operatorname{sen} x\cos x + \sqrt{3}a^2\cos^2 x$$

$$Q(x) = \sqrt{3}a^2\cos x\left(\sqrt{3}\operatorname{sen} x + \cos x\right) = a^2\left(3\operatorname{sen} x\cos x + \sqrt{3}\cos^2 x\right)$$

?

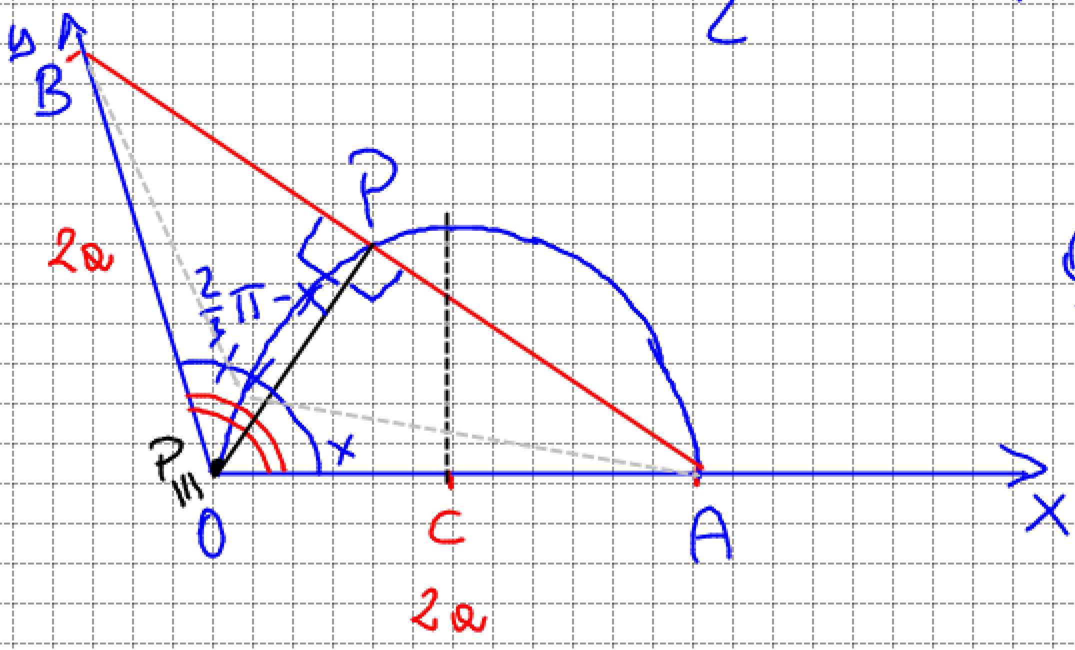
se $x = 0°$ $\quad Q(0) = \sqrt{3}a^2$

se $x = 45°$ $\quad Q(45°) = \sqrt{3}a^2\frac{\sqrt{2}}{2}\left(\sqrt{3}\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) =$

$$= \frac{\sqrt{6}}{2}a^2\left(\sqrt{3}+1\right)\frac{\sqrt{2}}{2} =$$

$$= \frac{\sqrt{3}}{2}a^2\left(\sqrt{3}+1\right)$$

(*)



$\overline{OA} = 2a$

$\overline{PO} = 2a\cos x$

$\widehat{POA} = \frac{\pi}{3}$

$0 \le x \le \frac{\pi}{3}$ $\qquad$ se $x = \frac{\pi}{3}$ $\qquad Q\left(\frac{\pi}{3}\right) = \sqrt{3}a^2\,\frac{1}{2}\left(\frac{3}{2} + \frac{1}{2}\right) = \sqrt{3}a^2$

$$Q(x) = \sqrt{3}a^2\cos x\left(\sqrt{3}\operatorname{sen} x + \cos x\right) =$$

$$= a^2\left[3\operatorname{sen} x\cos x + \sqrt{3}\cos^2 x\right] =$$

$$= a^2\left[\frac{3}{2}\operatorname{sen} 2x + \frac{\sqrt{3}}{2}\cos 2x + \frac{\sqrt{3}}{2}\right]$$

$\operatorname{sen} 2x = 2\operatorname{sen} x\cos x$

$\cos 2x = \cos^2 x - \operatorname{sen}^2 x = 2\cos^2 x - 1$

$\rightarrow \operatorname{sen} x\cos x = \frac{\operatorname{sen} 2x}{2}$

$\rightarrow \cos^2 x = \frac{1 + \cos 2x}{2}$

$$a\,\text{sen}\,\alpha + b\cos\alpha = \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\,\text{sen}\,\alpha + \frac{b}{\sqrt{a^2+b^2}}\cos\alpha\right)$$

$$= a^2\left[\frac{3}{2}\,\text{sen}\,2x + \frac{\sqrt{3}}{2}\cos 2x + \frac{\sqrt{3}}{2}\right] = a^2\left[\sqrt{\frac{9}{4}+\frac{3}{4}}\left(\frac{\frac{3}{2}}{\sqrt{3}}\,\text{sen}\,2x + \frac{1}{2}\cos 2x\right) +\right.$$

$$\left. + \frac{\sqrt{3}}{2}\right] = a^2\left[\sqrt{3}\left(\underbrace{\frac{\sqrt{3}}{2}}_{\cos\frac{\pi}{6}}\,\text{sen}\,2x + \underbrace{\frac{1}{2}}_{\text{sen}\frac{\pi}{6}}\cos 2x\right)\right] + \frac{\sqrt{3}}{2}a^2 =$$

$$Q(x) = a^2\left[\sqrt{3}\,\text{sen}\left(2x+\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}\right]$$

$$Q\left(\frac{\pi}{3}\right) = \sqrt{3}\,a^2$$

$$Q\left(\frac{\pi}{6}\right) = \frac{3}{2}\sqrt{3}\,a^2$$