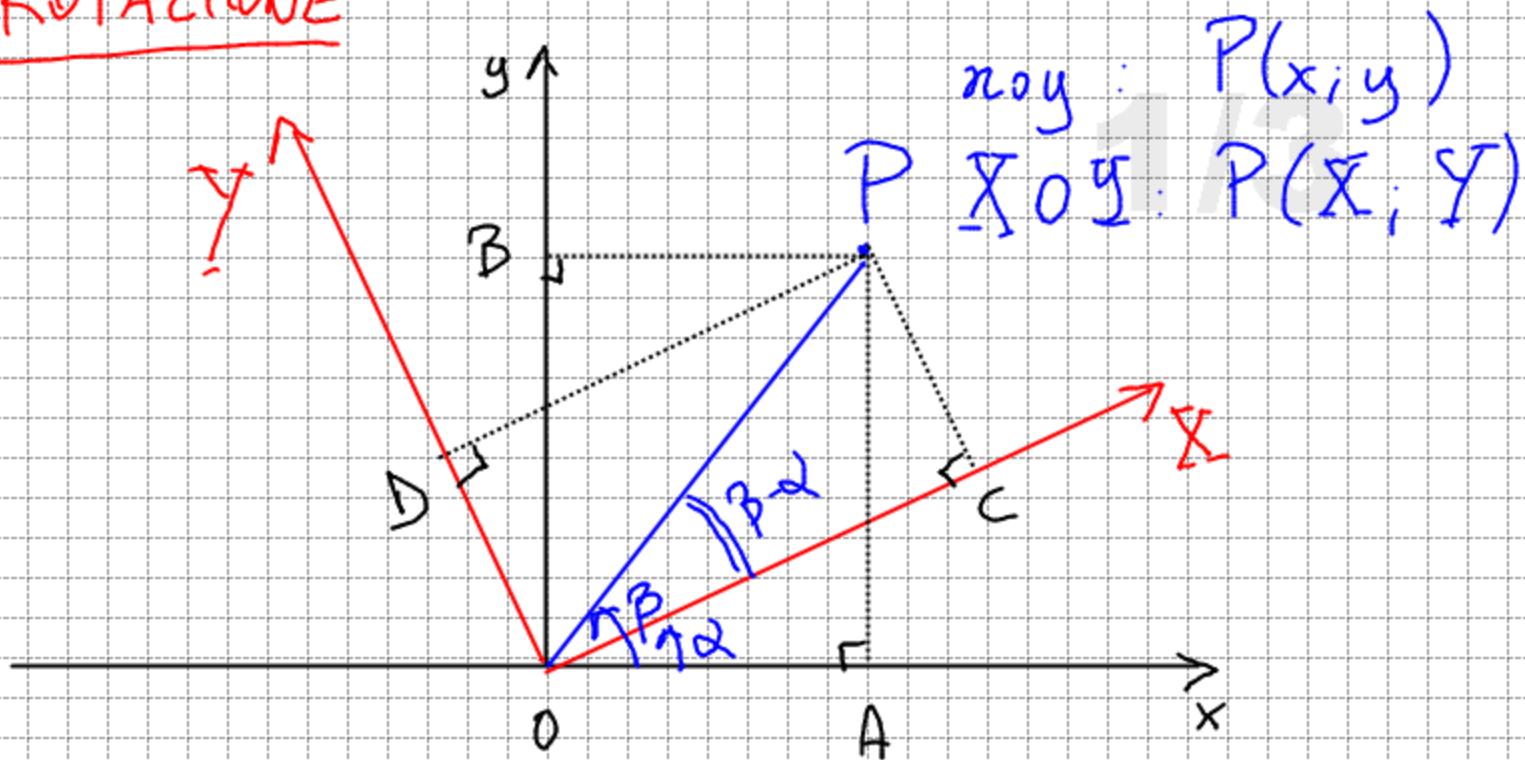


ROTAZIONE



$$x = \overline{OA} = \overline{BP} = \overline{OP} \cos \beta \quad y = \overline{OB} = \overline{AP} = \overline{OP} \sin \beta$$
$$\underline{X} = \overline{OC} = \overline{DP} = \overline{OP} \cos(\beta - \alpha) \quad \underline{Y} = \overline{OD} = \overline{CP} = \overline{OP} \sin(\beta - \alpha)$$

$$\underline{X} = \overline{OP} (\cos \beta \cos \alpha + \sin \beta \sin \alpha) = (\overline{OP} \cos \beta) \cos \alpha + (\overline{OP} \sin \beta) \sin \alpha$$

$$\underline{X} = x \cos \alpha + y \sin \alpha$$

$$\underline{Y} = \overline{OP} (\sin \beta \cos \alpha - \cos \beta \sin \alpha) = (\overline{OP} \sin \beta) \cos \alpha - (\overline{OP} \cos \beta) \sin \alpha$$

$$\underline{Y} = y \cos \alpha - x \sin \alpha$$

$$R: \begin{cases} \underline{X} = x \cos \alpha + y \sin \alpha \\ \underline{Y} = -x \sin \alpha + y \cos \alpha \end{cases}$$

$$R^{-1}: \begin{cases} x = \underline{X} \cos \alpha - \underline{Y} \sin \alpha \\ y = \underline{X} \sin \alpha + \underline{Y} \cos \alpha \end{cases}$$

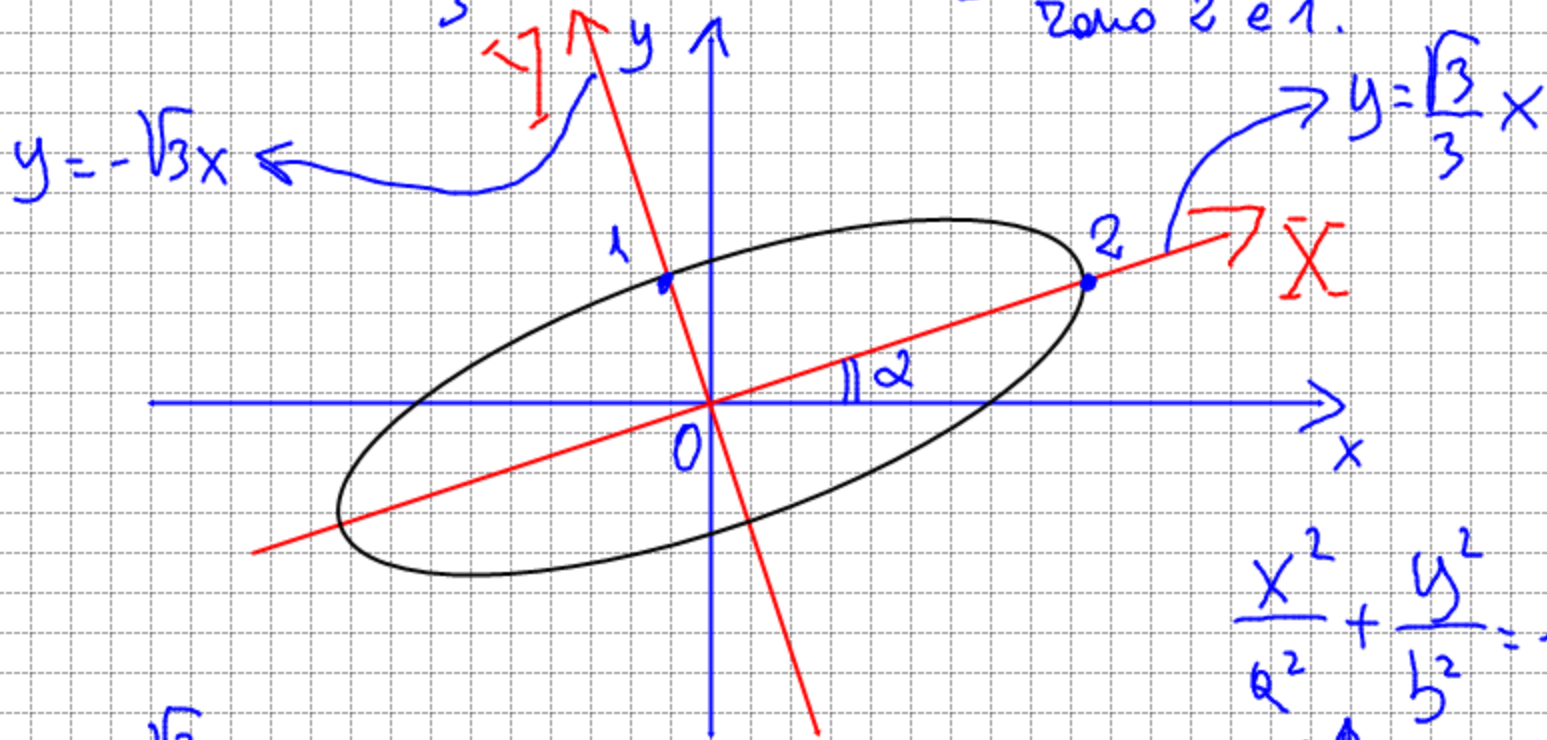


ESEMPIO

Dato l'ellisse i cui assi di simmetria sono:

$$X: y = \frac{\sqrt{3}}{3}x$$

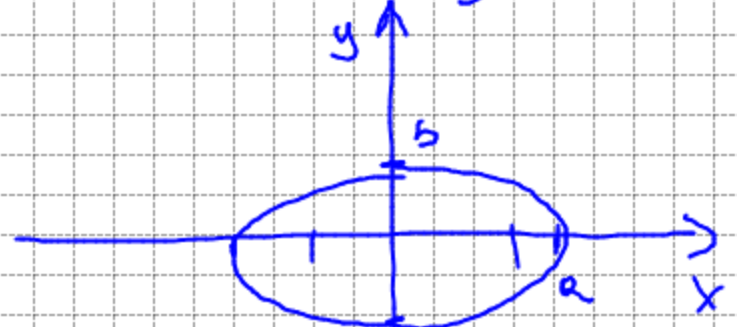
$$Y: y = -\sqrt{3}x \text{ e i semiasse misure } 2 \text{ e } 1.$$



$$\frac{\sqrt{3}}{3} = \text{Tg} \alpha \quad \alpha = 30^\circ$$

$$\frac{X^2}{4} + \frac{Y^2}{1} = 1$$

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$$



$$y = mx + q \quad m = \text{Tg} \alpha$$

$$\begin{cases} X = x \cos 30^\circ + y \text{ sen } 30^\circ \\ Y = -x \text{ sen } 30^\circ + y \cos 30^\circ \end{cases}$$

$$\Rightarrow \begin{cases} X = \frac{\sqrt{3}}{2}x + \frac{1}{2}y \\ Y = -\frac{1}{2}x + \frac{\sqrt{3}}{2}y \end{cases}$$

$$\frac{\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}y\right)^2}{4} + \frac{\left(-\frac{1}{2}x + \frac{\sqrt{3}}{2}y\right)^2}{1} = 1$$

$$\frac{3}{4}x^2 + \frac{1}{4}y^2 + \frac{\sqrt{3}}{2}xy + 4\left(\frac{1}{4}x^2 + \frac{3}{4}y^2 - \frac{\sqrt{3}}{2}xy\right) = 4$$

$$\frac{3}{4}x^2 + \frac{1}{4}y^2 + \frac{\sqrt{3}}{2}xy + x^2 + 3y^2 - 2\sqrt{3}xy - 4 = 0$$

$$\frac{7}{4}x^2 + \frac{13}{4}y^2 - \frac{3\sqrt{3}}{2}xy - 4 = 0$$

$$7x^2 + 13y^2 - 6\sqrt{3}xy - 16 = 0$$

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$$5x^2 - 6xy + 5y^2 - 1 = 0$$

$$R': \begin{cases} x = X \cos \alpha - Y \sin \alpha \\ y = X \sin \alpha + Y \cos \alpha \end{cases}$$

$$\begin{aligned} a &= \cos \alpha \\ b &= \sin \alpha \end{aligned}$$

$$\begin{cases} x = aX - bY \\ y = bX + aY \end{cases} \quad \boxed{a^2 + b^2 = 1}$$

$$5(aX - bY)^2 - 6(aX - bY)(bX + aY) + 5(bX + aY)^2 - 1 = 0$$

$$5(a^2X^2 + b^2Y^2 - 2abXY) - 6(abX^2 + a^2XY - b^2XY - abY^2) + 5(b^2X^2 + a^2Y^2 + 2abXY) - 1 = 0$$

$$5a^2X^2 + 5b^2Y^2 - 10abXY - 6abX^2 - 6(a^2 - b^2)XY + 6abY^2 + 5b^2X^2 + 5a^2Y^2 + 10abXY - 1 = 0$$

$$(5a^2 - 6ab + 5b^2)X^2 + (5b^2 + 6ab + 5a^2)Y^2 - 6(a^2 - b^2)XY - 1 = 0$$

$$\begin{cases} a^2 - b^2 = 0 \\ a^2 + b^2 = 1 \end{cases} \quad \begin{cases} a^2 = b^2 \\ b = \pm \frac{\sqrt{2}}{2} \end{cases} \quad a = \pm \frac{\sqrt{2}}{2}$$

$$\frac{X^2}{A^2} + \frac{Y^2}{B^2} = 1$$