INTEGRALE INDEFINITO

PRIMITIVA DI UNA FUNZIONE

- Sia $y=f(x)$, $y=F(x)+k$ $\forall k\in\mathbb{R}$ la famiglia di primitive di $y=f(x)$ se $F'(x)=f(x)$.

- Si dice integrale indefinito di $y=f(x)$:

funzione integranda $\leftarrow$ $\int f(x)\,dx$ $\rightarrow$ integrazione indefinita

la totalità delle primitive della funzione $y=f(x)$

ESEMPIO

- $f(x)=3$

$$\int 3\,dx = 3x+k \quad k\in\mathbb{R} \qquad D(3x+k)=3$$

- $f(x)=x$

$$\int x\,dx = \frac{1}{2}x^2+k \quad k\in\mathbb{R} \qquad D\left(\frac{1}{2}x^2+k\right)=\frac{2}{2}x=x$$

- $f(x)=\operatorname{sen} x$

$$\int \operatorname{sen} x\,dx = -\cos x+k \quad k\in\mathbb{R} \qquad D(-\cos x+k)=-(-\operatorname{sen} x)=\operatorname{sen} x$$

- $f(x)=\cos x$

$$\int \cos x\,dx = \operatorname{sen} x+k \quad k\in\mathbb{R} \qquad D(\operatorname{sen} x+k)=\cos x$$

- $f(x)=e^x$

$$\int e^x\,dx = e^x+k \quad k\in\mathbb{R} \qquad D(e^x+k)=e^x$$

- $f(x)=\frac{1}{|x|}$

$$\int \frac{1}{|x|}\,dx = \ln|x|+k \quad k\in\mathbb{R} \qquad D(\ln|x|+k)=\frac{1}{|x|}$$

PROPRIETÀ DELL'INTEGRALE INDEFINITO

1) $\int k\, f(x)\, dx = k \int f(x)\, dx \qquad k \in \mathbb{R}$

2) $\int \left[f_1(x) + f_2(x) + \dots + f_n(x) \right] dx = \int f_1(x)\, dx + \int f_2(x)\, dx + \dots + \int f_n(x)\, dx$

- $\int (x + x^2 + x^3)\, dx = \int x\, dx + \int x^2\, dx + \int x^3\, dx = \frac{1}{2} x^2 + \frac{1}{3} x^3 + \frac{1}{4} x^4 + c$

INTEGRALI IMMEDIATI

- $\int x^n\, dx = \frac{1}{n+1} x^{n+1} + k \qquad k \in \mathbb{R} \quad (n \neq -1)$
- $\int \frac{1}{x}\, dx = \ln|x| + k \qquad k \in \mathbb{R}$
- $\int \operatorname{sen} x\, dx = -\cos x + k \qquad k \in \mathbb{R}$
- $\int \cos x\, dx = \operatorname{sen} x + k \qquad k \in \mathbb{R}$
- $\int \frac{1}{\cos^2 x}\, dx = \operatorname{tg} x + k \qquad k \in \mathbb{R}$
- $\int \frac{1}{\operatorname{sen}^2 x}\, dx = -\operatorname{ctg} x + k \qquad k \in \mathbb{R}$
- $\int e^x\, dx = e^x + k; \; k \in \mathbb{R}$
- $\int a^x\, dx = a^x \log_a e + k; \; k \in \mathbb{R}$
- $\int \frac{1}{\sqrt{1-x^2}}\, dx = \operatorname{arcsen} x + k$
- $\int \frac{1}{1+x^2}\, dx = \operatorname{arctg} x + k$

REGOLE DI INTEGRAZIONE DELLE FUNZIONI COMPOSTE

$$\int [f(x)]^{\alpha} \underbrace{f'(x)\,dx}_{d(f(x))} = \frac{[f(x)]^{\alpha+1}}{\alpha+1} + k \qquad k \in \mathbb{R} \quad \alpha \neq -1$$

ESEMPIO

$$\int (3x)^{5} \underbrace{3\,dx}_{d(3x)} = \frac{(3x)^{5+1}}{5+1} + k \qquad k \in \mathbb{R}$$

$k \in \mathbb{R}$

- $\displaystyle\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + k$
- $\displaystyle\int f'(x)\,\operatorname{sen} f(x)\,dx = -\cos f(x) + k$
- $\displaystyle\int f'(x)\cos f(x)\,dx = \operatorname{sen} f(x) + k$
- $\displaystyle\int \frac{f'(x)}{\cos^2 f(x)}\,dx = \operatorname{Tg}(f(x)) + k$

$\displaystyle\int \frac{f'(x)}{\operatorname{sen}^2 f(x)}\,dx = -\operatorname{cTg} f(x) + k$

$\displaystyle\int f'(x)\,e^{f(x)}\,dx = e^{f(x)} + k$

$\displaystyle\int f'(x)\,a^{f(x)}\,dx = a^{f(x)} \log_a e + k$

$\displaystyle\int \frac{f'(x)}{\sqrt{1-[f(x)]^2}}\,dx = \operatorname{arcsen} f(x) + k$

$\displaystyle\int \frac{f'(x)}{1+[f(x)]^2}\,dx = \operatorname{arcTg} f(x) + k$

ESEMPI

- $\displaystyle -\int \underbrace{(-\operatorname{sen} x)}_{f'(x)}\, \cos^4 \underbrace{x}_{f(x)}\,dx = -\frac{[\cos x]^5}{5} + k \qquad k \in \mathbb{R}$
- $\displaystyle \int \underbrace{2}_{f'(x)} \sqrt{\underbrace{2x+1}_{f(x)}}\,dx = \int (2x+1)^{\frac{1}{2}}\,d(2x+1) = \frac{(2x+1)^{\frac{1}{2}+1}}{\frac{3}{2}} + k$

ESERCIZI

$\displaystyle\int \operatorname{Tg} x\,dx$; $\displaystyle\int \frac{1}{2x+1}\,dx$; $\displaystyle\int \cos 2x\,dx$; $\displaystyle\int \frac{1}{\cos^2 3x}\,dx$;

$\displaystyle\int \frac{1}{\operatorname{sen}^2(3-x)}\,dx$