ESERCIZI

N 17 PAG 181 (Verificare l'identità)

$$\binom{n}{3} - 2\binom{n+1}{3} + \binom{n+2}{3} = n \qquad n \geq 3$$

$$\frac{n!}{3!\,(n-3)!} - 2\,\frac{(n+1)!}{3!\,(n+1-3)!} + \frac{(n+2)!}{3!\,(n+2-3)!} = n$$

$$\frac{n!}{3!\,(n-3)!} - 2\,\frac{(n+1)!}{3!\,(n-2)!} + \frac{(n+2)!}{3!\,(n-1)!} = n$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{3!\,\cancel{(n-3)!}} - \frac{2(n+1)(n)(n-1)\cancel{(n-2)!}}{3!\,\cancel{(n-2)!}} +$$

$$+ \frac{(n+2)(n+1)\,n\,\cancel{(n-1)!}}{3!\,\cancel{(n-1)!}} = n$$

$$\frac{n(n-1)(n-2) - 2n(n+1)(n-1) + n(n+1)(n+2)}{6} = n$$

$$\frac{n\left[\cancel{n^2} - \cancel{2n} - \cancel{n} + 2 - \cancel{2n^2} + \cancel{2n} - \cancel{2n} + 2 + \cancel{n^2} + \cancel{2n} + \cancel{n} + 2\right]}{6} = n$$

$$n\,\frac{\cancel{6}}{\cancel{6}} = n \qquad n = n$$

N 35

$$\binom{m}{k} = \binom{m-2}{k} + 2\binom{m-2}{k-1} + \binom{m-2}{k-2} \quad \text{con} \quad \begin{matrix} m \geq 4 \\ k \geq 2 \end{matrix}$$

$$\left.\begin{matrix} m \geq k\,; & k \geq 0 & m \geq 0 \\ m-2 \geq k & m-2 \geq 0 & k \geq 0 \\ m-2 \geq k-1 & m-2 \geq 0 & k-1 \geq 0 \\ m-2 \geq k-2 & m-2 \geq 0 & k-2 \geq 0 \end{matrix}\right\} \Rightarrow \begin{matrix} k \geq 2 \\ m \geq 4. \end{matrix}$$

$$\frac{m!}{k!\,(m-k)!} = \underbrace{\frac{(m-2)!}{k!\,(m-2-k)!}} + 2\underbrace{\frac{(m-2)!}{(k-1)!\,(m-k-1)!}} + \underbrace{\frac{(m-2)!}{(k-2)!\,(m-k)!}}$$

$$\frac{m!}{k!\,(m-k)!} = \frac{(m-k)(m-k-1)(m-2)! + 2(m-2)!\,(k)(m-k) + (m-2)!\,k(k-1)}{k!\,(m-k)!}$$

$$\frac{m!}{k!\,(m-k)!} = \frac{(m-2)!}{k!\,(m-k)!}\left[(m-k)(m-k-1) + 2k(m-k) + k(k-1)\right]$$

$$\downarrow$$

$$m^2 - \cancel{km} - m - \cancel{km} + \cancel{k^2} + \cancel{k} + \cancel{2km} - \cancel{2k^2} + \cancel{k^2} - \cancel{k}$$

$$\parallel$$

$$m(m-1)$$

$$(m-2)! \cdot m(m-1) = m!$$