problema 2 anno 2012/13

$f(x)=\dfrac{8}{4+x^2}$ $\quad D_f=\{x\in\mathbb{R} / 4+x^2\neq 0\}=$

$=\{x\in\mathbb{R}\}=(-\infty;+\infty)$

Segni

$\dfrac{8}{4+x^2}>0$ $\quad f(x)$ sempre positiva

Intersezione con assi

$\begin{cases}\dfrac{8}{4+x^2}=0\\ y=0\end{cases}$ Non si hanno intesezioni con asse x

$\begin{cases}y=\dfrac{8}{4+x^2}\\ x=0\end{cases}$ $\begin{cases}y=2\\ x=0\end{cases}$ $A(0;2)$

Limiti e asintoti

$\lim\limits_{x\to-\infty}\dfrac{8}{4+x^2}=\left[\dfrac{8}{4+(-\infty)^2}=\dfrac{8}{+\infty}\right]=0^+$

$\lim\limits_{x\to+\infty}\dfrac{8}{4+x^2}=\left[\dfrac{8}{+\infty}\right]=0^+$ $\quad y=0$ Asintoto orizzontale

Simmetrie

$f(-x)=\dfrac{8}{4+(-x)^2}=\dfrac{8}{4+x^2} \Rightarrow f(x)=f(-x)$ Pari

La funzione e' simmetrica rispetto l'asse y

Derivata, Massimi e Minimi

$f(x)=\dfrac{8}{4+x^2}$ $\quad f'(x)=\dfrac{-8(2x)}{(4+x^2)^2}=\dfrac{-16x}{(4+x^2)^2}$

$\dfrac{-16x}{(4+x^2)^2}\geq 0$ $\quad N\geq 0$: $-16x\geq 0$, $x\leq 0$ $\quad D>0$: $(4+x^2)^2>0$ sempre positivo

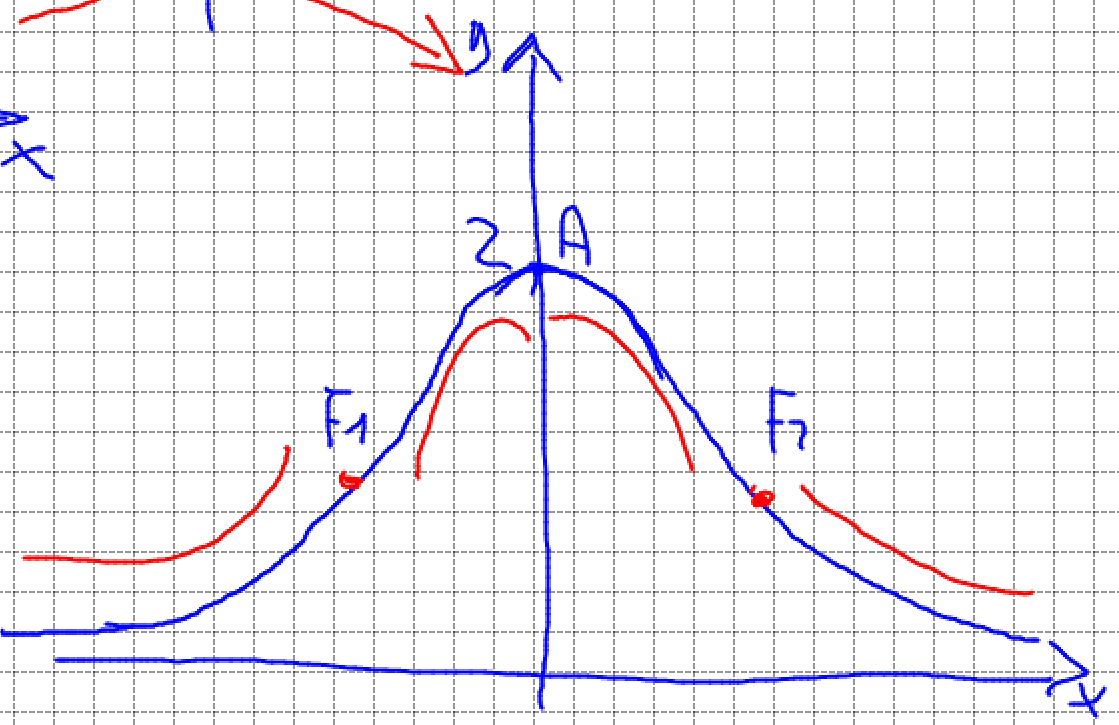
		0	
N	+	●	−
D	+		+
f'(x)	+	●	−

$x=0;\ y=2$

$A(0;2)$ massimo

		0		
f(x)	+ +		+ +	x
f'(x)	+	A ●	—	

y=0 ASINTOTO ORIZZONTALE



$$f''(x) = D\left(\frac{-16x}{(4+x^2)^2}\right) = \frac{-16(4+x^2)^2 + 16x\,(2(4+x^2)2x)}{(4+x^2)^4} =$$

$$= \frac{+16\,\cancel{(4+x^2)}\left[-4-x^2+4x^2\right]}{(4+x^2)^{\cancel{4}\,3}} = \frac{16\left[3x^2-4\right]}{(4+x^2)^3}$$

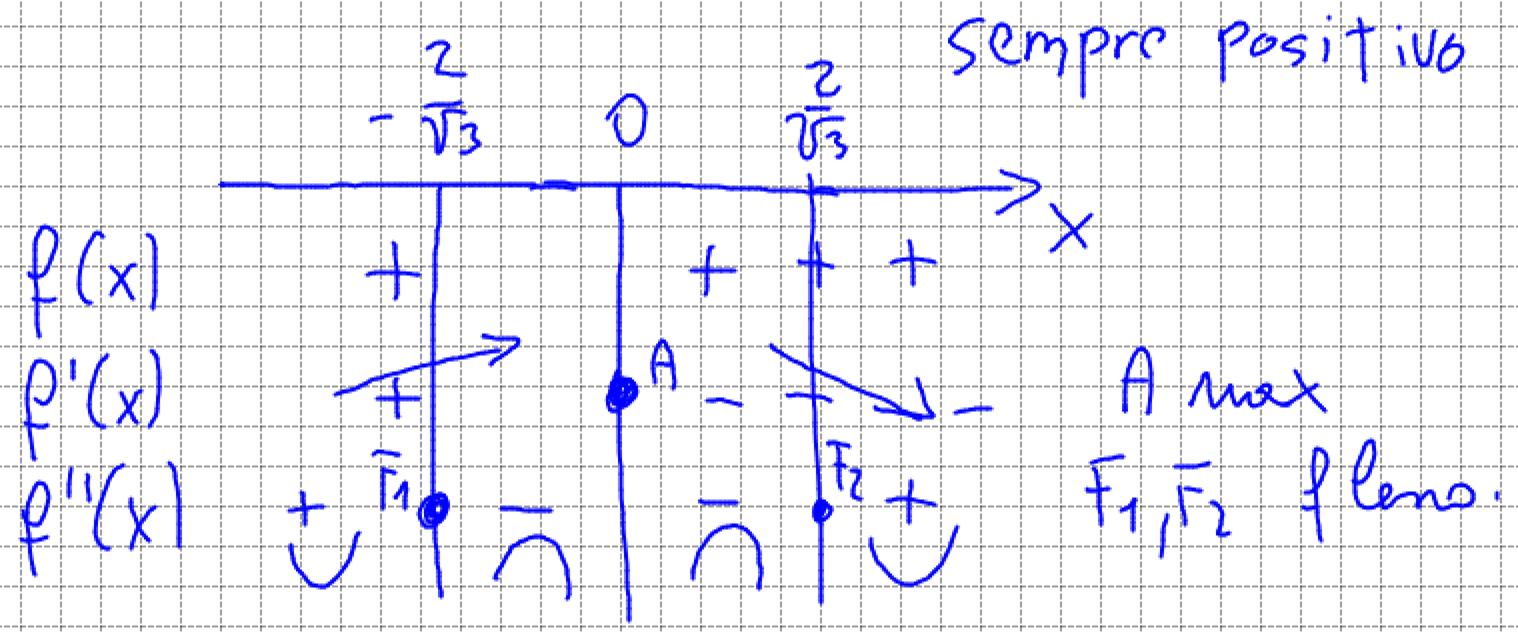
$$f''(x) \geq 0;\quad \frac{16(3x^2-4)}{(4+x^2)^3} \geq 0$$

N $\quad 3x^2-4 \geq 0$

$x \leq -\frac{2}{\sqrt{3}} \cup x \geq +\frac{2}{\sqrt{3}}$

D $\quad (4+x^2)^3 > 0$

sempre positivo



$P(-2;1)\quad Q(2;1)$ trovare l'equazioni delle tangenti alla curva passanti per P e Q

$$f'(x) = \frac{-16x}{(4+x^2)^2}$$

$$f'(-2) = \frac{32}{64} = \frac{1}{2}$$

$$f'(2) = \frac{-32}{64} = -\frac{1}{2}$$

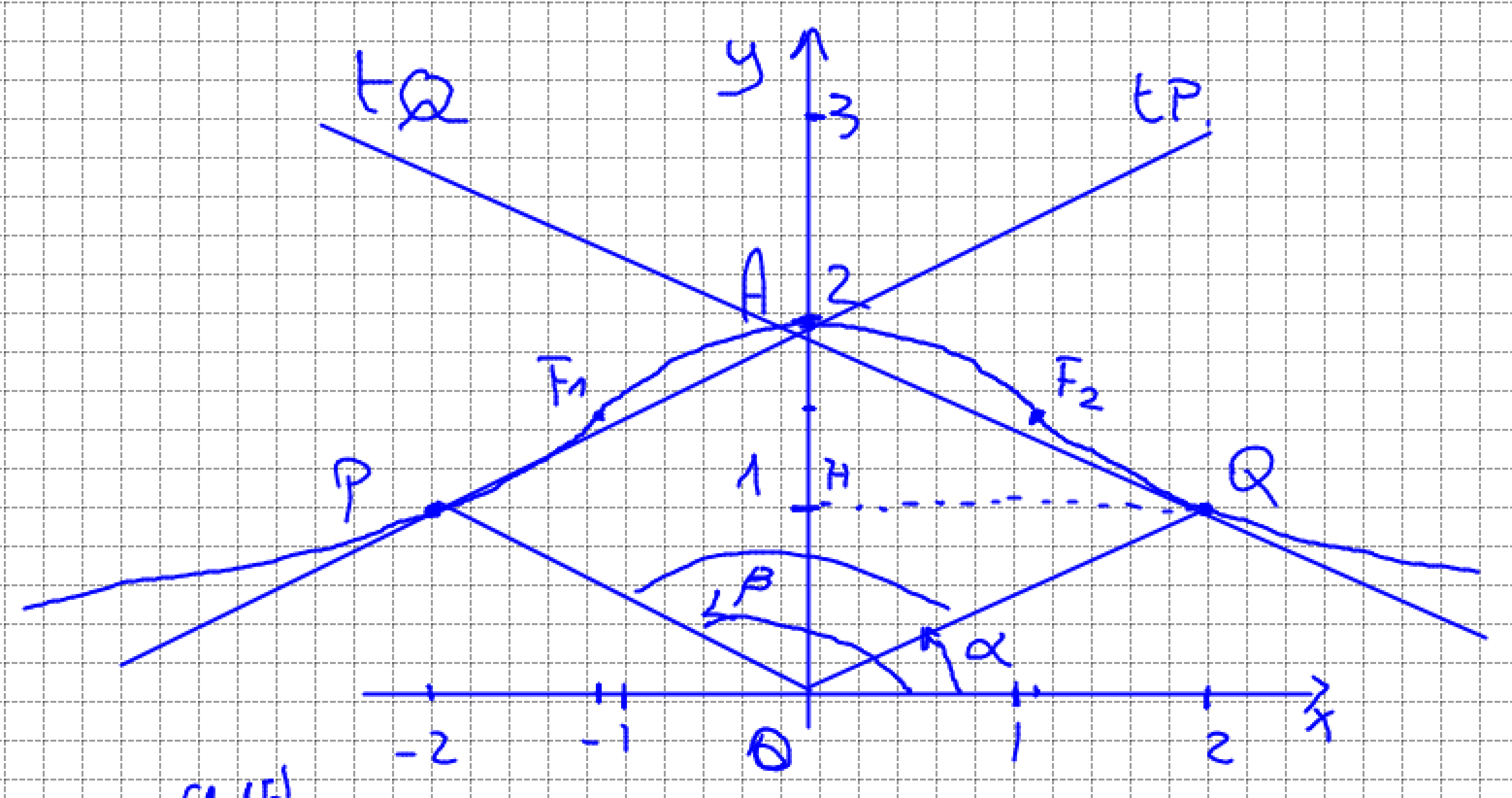
$y - y_P = f'(-2)(x - x_P)$

$y - 1 = \frac{1}{2}(x+2)$

$y = \frac{1}{2}x + 2$

$y - 1 = -\frac{1}{2}(x-2)$

$y = -\frac{1}{2}x + 2$



$(-1{,}15)$

$$F_1\left(-\frac{2}{\sqrt{3}};\ \frac{3}{2}\right) \qquad \frac{8}{4+\frac{4}{3}} = \frac{24}{16} = \frac{3}{2}$$

$$F_2\left(\frac{2}{\sqrt{3}};\ \frac{3}{2}\right)$$

$P(-2;1) \qquad y = \frac{1}{2}x + 2\ (tP)$

x	y
0	2
2	3

$Q(2;1) \qquad y = -\frac{1}{2}x + 2\ (tQ)$

x	y
0	2
2	1

$P(-2;1)$
$A(0;2)$
$Q(2;1)$
$O(0;0)$

$$PA=\sqrt{(x_P-x_A)^2+(y_P-y_A)^2}=$$
$$\sqrt{4+1}=\sqrt{5}$$

$$OP=\sqrt{(x_O-x_P)^2+(y_O-y_P)^2}=$$
$$=\sqrt{4+1}=\sqrt{5}$$

$$PA=OP=\sqrt{5}$$

$$OQ=\sqrt{(x_O-x_Q)^2+(y_O-y_Q)^2}$$
$$=\sqrt{4+1}=\sqrt{5}$$

$$QA=\sqrt{(x_Q-x_A)^2-(y_Q-y_A)^2}$$
$$=\sqrt{4+1}=\sqrt{5}$$

non é un quadrato perché le diagonali non sono uguali, quindi é un rombo

la retta che passa per OQ é parallela alla retta che passa per PA e quindi $m_{OQ}=m_{PA}=\frac{1}{2}$

quind $tg_\alpha=\frac{1}{2} \Rightarrow \alpha=arctg\frac{1}{2}$,
$\alpha = 26°33'54''$

$$\beta=(90°-26°33'54'')\cdot 2=$$
$$=63°26'05''\cdot 2=126°52'11''$$

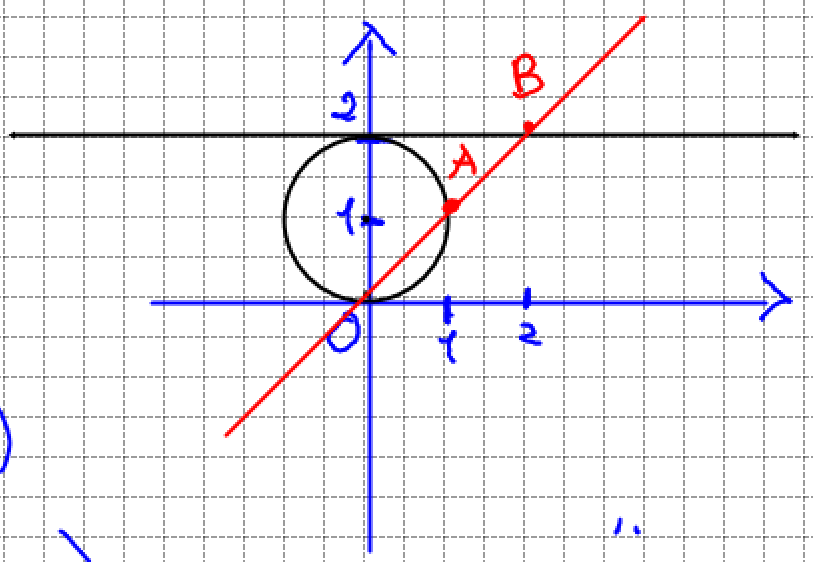
$$\mu=180°-126°52'11''=53°7'49''$$

D) $r=1$
$C=(0;1)$

$$\sqrt{(x-x_c)^2+(y-y_c)^2}=r$$

$$(x-0)^2+(y-1)^2=1$$
$$x^2+y^2\cancel{+1}-2y=\cancel{1}$$
$$x^2+y^2-2y=0$$

una retta t passante in O(0;0) taglia γ oltre che in O in un punto A e taglia $y=2$ in un punto B si trovi che qualunque sia t x di B ed y di A sono le coordine della funzione di partenza: $y=\frac{8}{4+x^2}$



$t: y=mx$

$2=mx$

$x=\frac{2}{m}$

$B(x;2)$

$B\left(\frac{2}{m};2\right)$

$$\begin{cases} x^2+y^2-2y=0 \\ y=mx \longrightarrow x=\frac{y}{m} \end{cases} \qquad \begin{cases} x=\frac{y}{m} \\ \frac{y^2}{m^2}+y^2-2y=0 \end{cases}$$

$$\begin{cases} x=\frac{y}{m} \\ y^2+m^2y^2-2m^2y=0 \end{cases}$$

$$y^2(1+m^2)-2m^2y=0$$
$$y(y(1+m^2)-2m^2)=0$$
$$y=0$$

$$A\begin{cases} y=+\frac{2m^2}{1+m^2} \\ x=\frac{2m}{1+m^2} \end{cases} \qquad A\left(\frac{2m}{1+m^2};\frac{2m^2}{1+m^2}\right)$$

$$y=\frac{8}{4+x^2} \qquad \frac{2m^2}{1+m^2}=\frac{8}{4+\frac{4}{m^2}}$$

$$\frac{2m^2\left(4+\frac{4}{m^2}\right)}{(1+m^2)\left(4+\frac{4}{m^2}\right)}=\frac{8(1+m^2)}{(1+m^2)\left(4+\frac{4}{m^2}\right)}$$

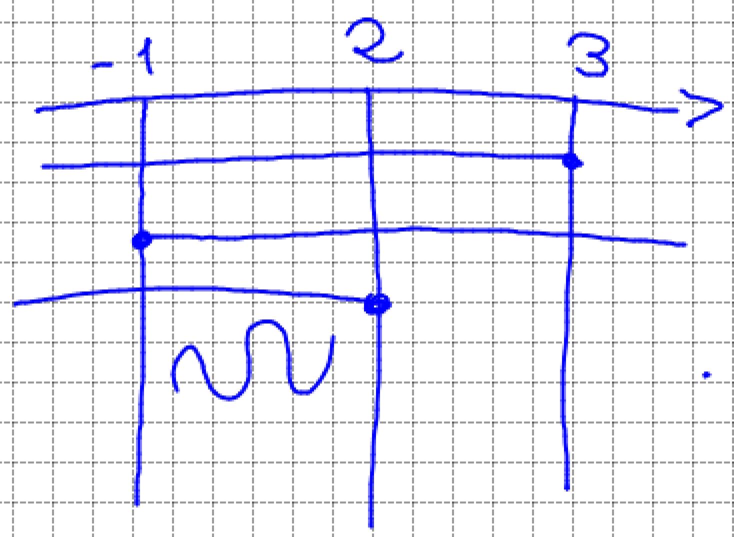
$$\frac{8m^2+8}{(1+m^2)\left(4+\frac{4}{m^2}\right)}=\frac{8+8m^2}{(1+m^2)\left(4+\frac{4}{m^2}\right)}$$

$$f(x) = \sqrt{1 - \sqrt{2 - \sqrt{3 - x}}}$$

$$D f : \forall x \in \mathbb{R} \Big/ \begin{cases} x \le 3 \\ 2 - \sqrt{3-x} \ge 0 \\ 1 - \sqrt{2 - \sqrt{3-x}} \ge 0 \end{cases}$$

$$\begin{cases} x \le 3 \\ 2 \ge \sqrt{3-x} \\ 1 \ge \sqrt{2 - \sqrt{3-x}} \end{cases} \qquad \begin{cases} x \le 3 \\ 4 \ge 3 - x \\ 1 \ge 2 - \sqrt{3-x} \end{cases}$$

$$\begin{cases} x \le 3 \\ x \ge -1 \\ 3 - x \ge 1 \end{cases} \qquad \begin{cases} x \le 3 \\ x \ge -1 \\ x \le 2 \end{cases}$$



$[-1; 2]$

$$f(x) = \operatorname{sen} \frac{3}{2} \pi x$$