problema 2 anno 2012/13

$f(x) = \frac{8}{5+x^2}$ $\quad D_f = \{x \in \mathbb{R} / 5+x^2 \neq 0\} =$

$= \{x \in \mathbb{R}\} = (-\infty; +\infty)$

Segni

$\frac{8}{5+x^2} > 0$ $\quad f(x)$ sempre positiva

Intersezione con assi

$\begin{cases} \frac{8}{5+x^2} = 0 \\ y = 0 \end{cases}$ Non si hanno intesezioni con asse x

$\begin{cases} y = \frac{8}{5+x^2} \\ x = 0 \end{cases}$ $\begin{cases} y = 2 \\ x = 0 \end{cases}$ $A(0;2)$

Limiti e asintoti

$\lim_{x \to -\infty} \frac{8}{5+x^2} = \left[\frac{8}{5+(-\infty)^2} = \frac{8}{+\infty}\right] = 0^+$

$\lim_{x \to +\infty} \frac{8}{5+x^2} = \left[\frac{8}{+\infty}\right] = 0^+$ $\quad y = 0$ Asintoto orizzontale

Simmetrie

$f(-x) = \frac{8}{5+(-x)^2} = \frac{8}{5+x^2} \Rightarrow f(x) = f(-x)$ Pari

La funzione é simmetrica rispetto l'asse y

Derivata, Massimi e Minimi

$f(x) = \frac{8}{5+x^2}$ $\quad f'(x) = \frac{-8(2x)}{(5+x^2)^2} = \frac{-16x}{(5+x^2)^2}$

$\frac{-16x}{(5+x^2)^2} \geq 0$ $\quad N \geq 0$ $\quad -16x \geq 0$ $\quad x \leq 0$ $\quad D > 0$ $\quad (5+x^2)^2 > 0$ sempre positivo

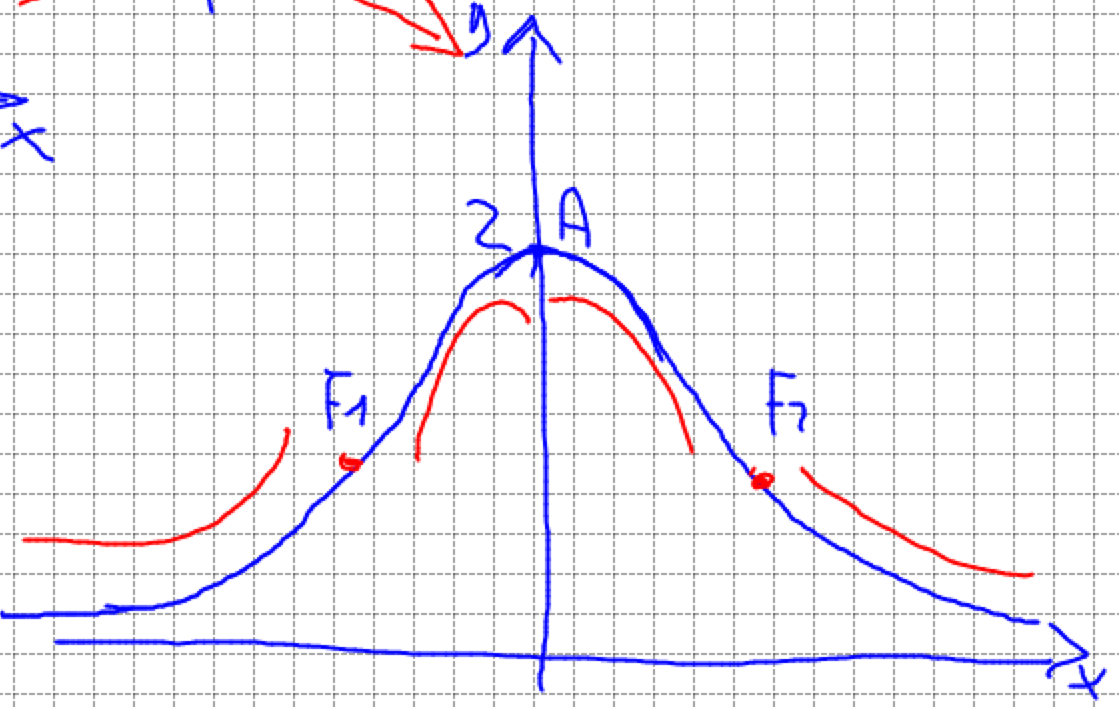
		0	
N	+	●	−
D	+		+
f'(x)	+	●	−

$x = 0;\ y = 2$

$A(0;2)$ massimo

		0		
f(x)	+	+	+	+
f'(x)	+	A ●	−	

$y = 0$ ASINTOTO ORIZZONTALE



$$f''(x) = D\left(\frac{-16x}{(4+x^2)^2}\right) = \frac{-16(4+x^2)^2 + 16x\left(2(4+x^2)2x\right)}{(4+x^2)^4} =$$

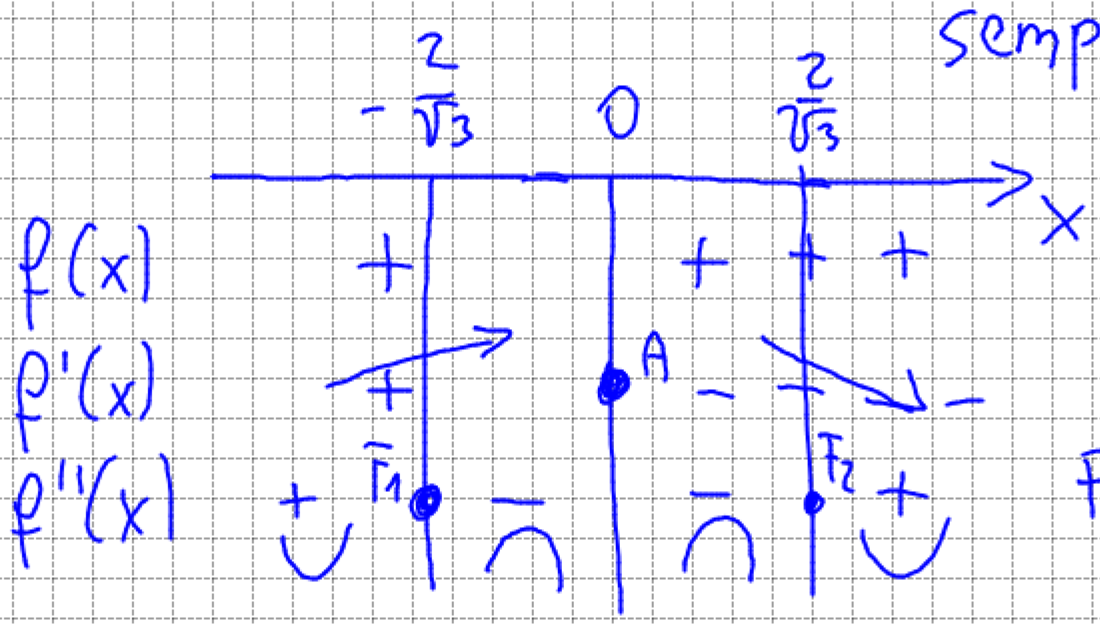
$$= \frac{+16\cancel{(4+x^2)}\left[-4-x^2+4x^2\right]}{(4+x^2)^{\cancel{4}3}} = \frac{16\left[3x^2-4\right]}{(4+x^2)^3}$$

$f''(x) \geq 0, \quad \frac{16(3x^2-4)}{(4+x^2)^3} \geq 0$ N $3x^2-4 \geq 0$

$x \leq -\frac{2}{\sqrt{3}} \cup x \geq +\frac{2}{\sqrt{3}}$

D $(4+x^2)^3 > 0$

sempre positivo



A max

F_1, F_2 flesso.

$P(-2;1)$ $Q(2;1)$ trovare l'equazioni delle tangenti alla curva passanti per P e Q

$$f'(x) = \frac{-16x}{(4+x^2)^2}$$

$$f'(-2) = \frac{32}{64} = \frac{1}{2}$$

$$f'(2) = \frac{-32}{64} = -\frac{1}{2}$$

$y - y_P = f'(-2)(x - x_P)$

$y - 1 = \frac{1}{2}(x+2)$

$y = \frac{1}{2}x + 2$

$y - 1 = -\frac{1}{2}(x-2)$

$y = -\frac{1}{2}x + 2$

finire per casa