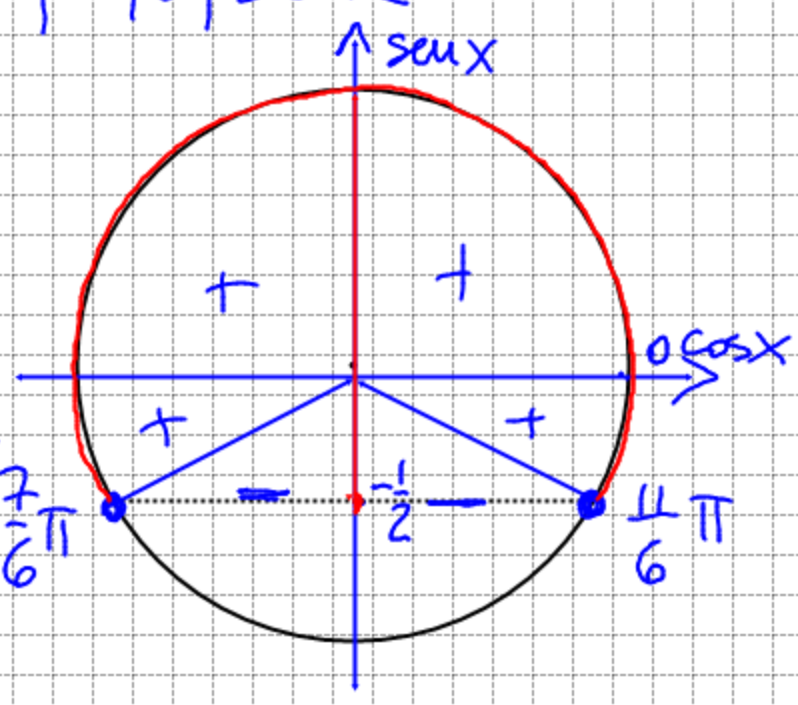
DISEQUAZIONI GONIOMETRICHE

ELEMENTARI

- $\sin x > m$ $(\sin x \geq m)$ oppure $\sin x < m$ $(\sin x \leq m)$
- $\cos x > m$ $(\cos x \geq m)$ = $\cos x < m$ $(\cos x \leq m)$
- $\operatorname{tg} x > t$ $(\operatorname{tg} x \geq t)$ = $\operatorname{tg} x < t$ $(\operatorname{tg} x \leq t)$
- $\operatorname{ctg} x > s$ $(\operatorname{ctg} x \geq s)$ = $\operatorname{ctg} x < s$ $(\operatorname{ctg} x \leq s)$

$m, n, t, s \in \mathbb{R}$

1) $\sin x \geq -\frac{1}{2}$



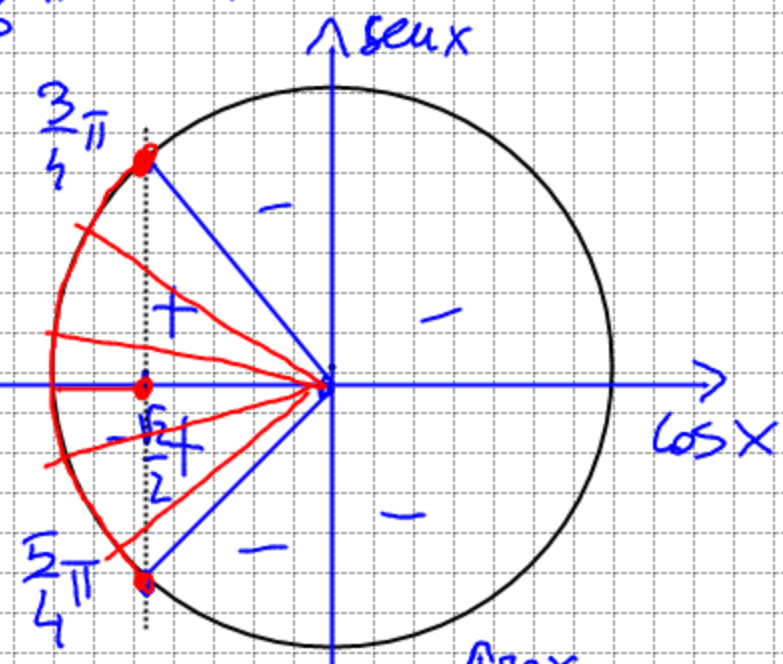
$S: 0 + 2k\pi \leq x \leq \frac{7}{6}\pi + 2k\pi$

$\cup$

$\frac{11}{6}\pi + 2k\pi \leq x \leq 2\pi + 2k\pi \quad k \in \mathbb{N}$

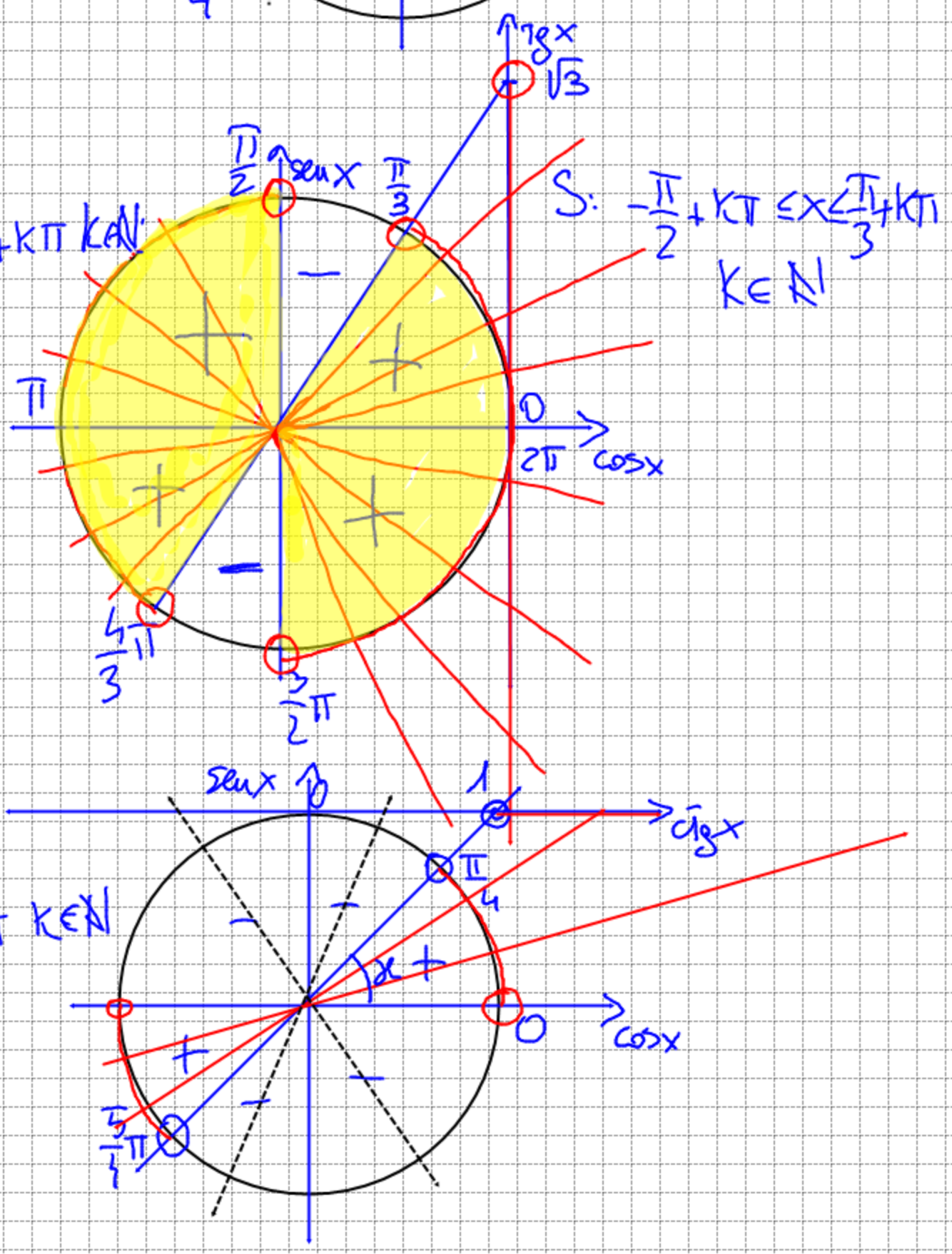
$S: -\frac{\pi}{6} + 2k\pi \leq x \leq \frac{7}{6}\pi + 2k\pi \quad k \in \mathbb{N}$

2) $\cos x \leq -\frac{\sqrt{2}}{2}$



$S: \frac{3}{4}\pi + 2k\pi \leq x \leq \frac{5}{4}\pi + 2k\pi \quad k \in \mathbb{N}$

3) $\operatorname{tg} x < \sqrt{3}$



$S: -\frac{\pi}{2} + k\pi < x < \frac{\pi}{3} + k\pi \quad k \in \mathbb{N}$

oppure

$S: 0 + k\pi \leq x < \frac{\pi}{3} + k\pi \cup \frac{\pi}{2} + k\pi < x < \pi + k\pi$ con $k \in \mathbb{N}$

$S: -\frac{\pi}{2} + k\pi \leq x < \frac{\pi}{3} + k\pi \quad k \in \mathbb{N}$

4) $\operatorname{ctg} x > 1$

$0 + k\pi < x < \frac{\pi}{4} + k\pi \quad k \in \mathbb{N}$

DISEQUAZIONI RICONDUCIBILI ALLE ELEMENTARI

1) $4\cos^2 x + 2(1-\sqrt{3})\cos x - \sqrt{3} > 0$ $\xrightarrow{\cos x = t}$ $4t^2 + 2(1-\sqrt{3})t - \sqrt{3} > 0$

$at^2+bt+c = a(t-t_1)(t-t_2) > 0$

$$\cos x = \frac{\sqrt{3}-1 \pm \sqrt{3+1-2\sqrt{3}+4\sqrt{3}}}{4} = \frac{\sqrt{3}-1 \pm \sqrt{(\sqrt{3}+1)^2}}{4} =$$

$$= \frac{\sqrt{3}-1 \pm (\sqrt{3}+1)}{4} \begin{cases} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{cases}$$

$\cos x = -\frac{1}{2}$ $\quad \cos x + \frac{1}{2} = 0$

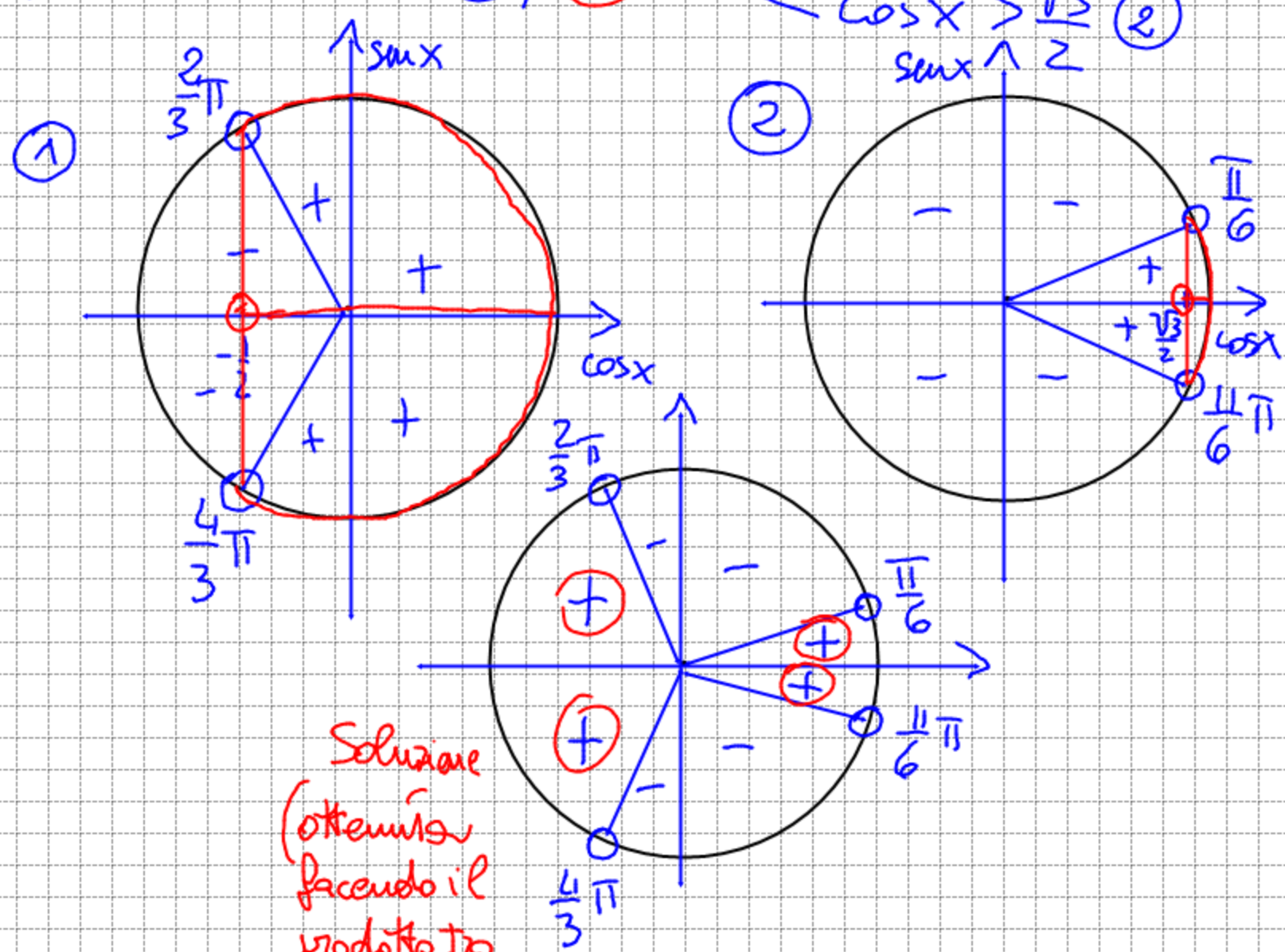
$\cos x = \frac{\sqrt{3}}{2}$ $\quad \cos x - \frac{\sqrt{3}}{2} = 0$

$$4\left(\cos x + \frac{1}{2}\right)\left(\cos x - \frac{\sqrt{3}}{2}\right) > 0$$

$\cos x > -\frac{1}{2}$ ①

$\cos x > \frac{\sqrt{3}}{2}$ ②

① ② Soluzione (ottenuta facendo il prodotto tra ① e ②).



$S: \; 0 + 2k\pi \le x < \frac{\pi}{6} + 2k\pi \cup \frac{2}{3}\pi + 2k\pi < x < \frac{4}{3}\pi + 2k\pi \cup$

$\cup \frac{11}{6}\pi + 2k\pi < x \le 2\pi + 2k\pi \quad \text{con } k \in \mathbb{N}$

oppure

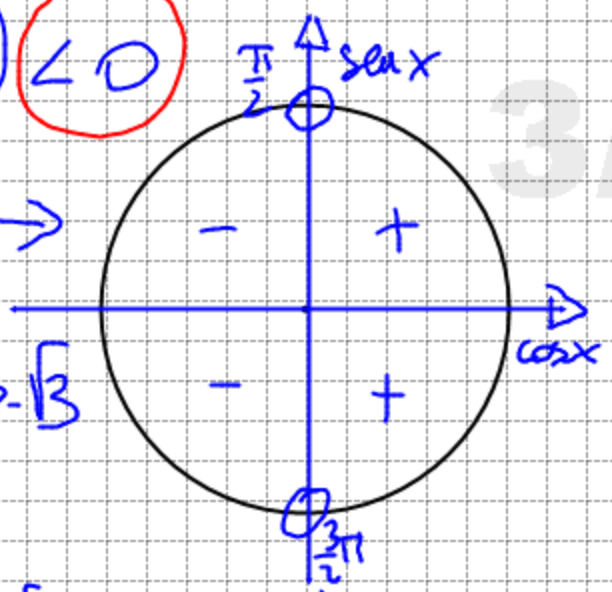
$-\frac{\pi}{6} + 2k\pi < x < \frac{\pi}{6} + 2k\pi \cup \frac{2}{3}\pi + 2k\pi < x < \frac{4}{3}\pi + 2k\pi$

con $k \in \mathbb{Z}$.

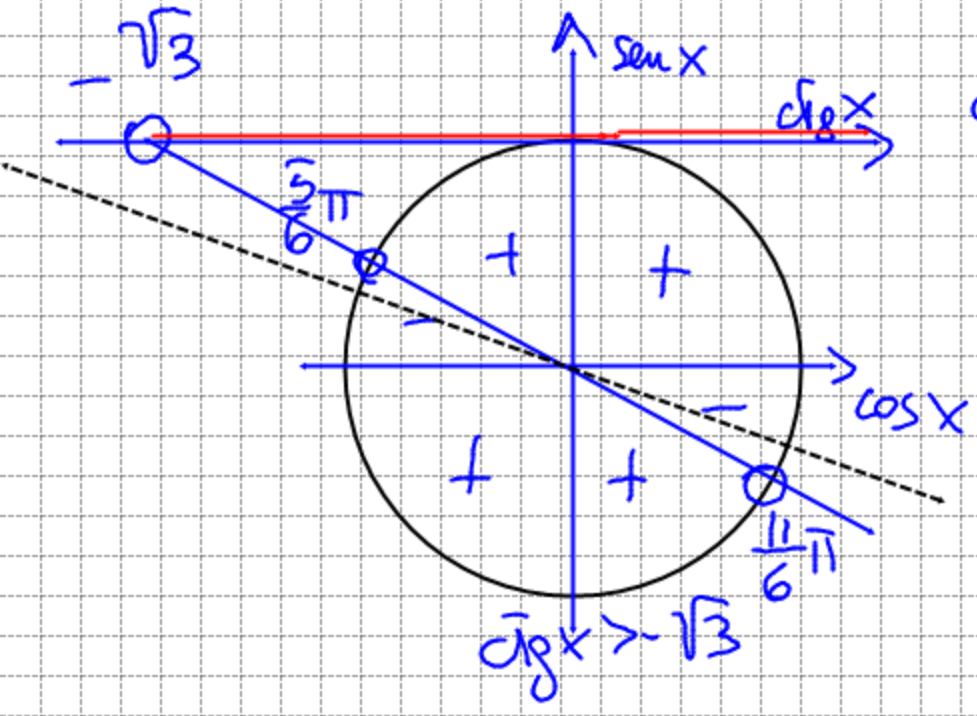
2) $\cos x\left(\operatorname{ctg} x+\sqrt{3}\right)<0$

$\cos x>0$

$\operatorname{ctg} x+\sqrt{3}>0 \quad \operatorname{ctg} x>-\sqrt{3}$

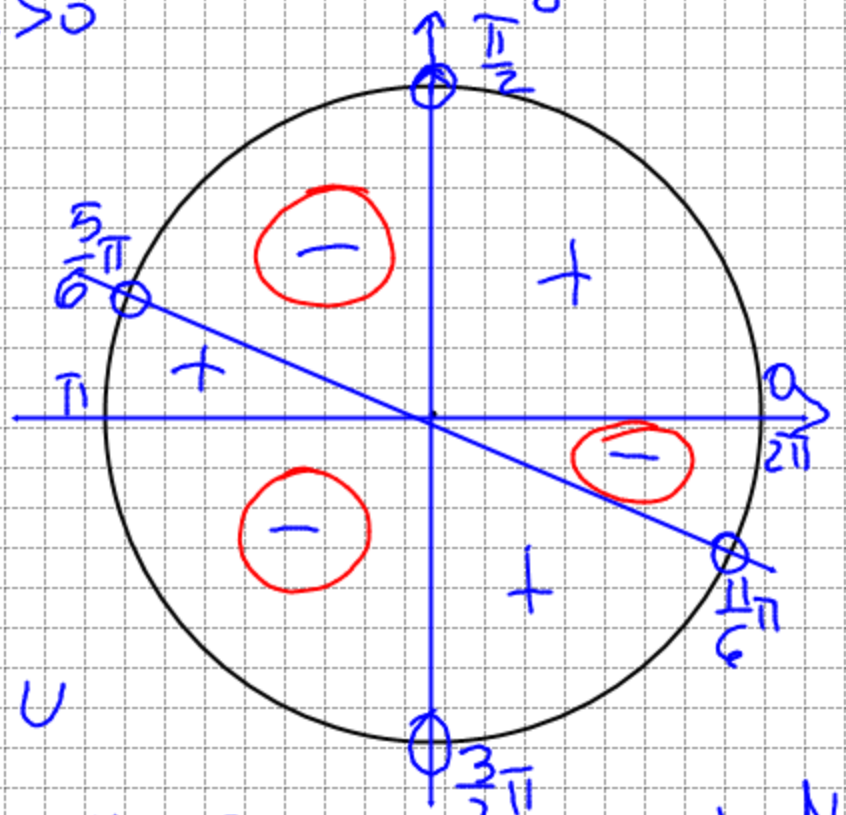


$\cos x>0$



$\operatorname{ctg} x>-\sqrt{3}$

$\cos x\left(\operatorname{ctg} x+\sqrt{3}\right)<0$



$$S:\ \frac{\pi}{2}+2k\pi<x<\frac{5}{6}\pi+2k\pi \cup$$

$$\cup\ \pi+2k\pi<x<\frac{3}{2}\pi+2k\pi \cup \frac{11}{6}\pi+2k\pi<x<2\pi+2k\pi \quad k\in\mathbb{N}$$