

DERIVATA DEL QUOZIENTE

$$y = \frac{f(x)}{g(x)} \quad D\left(\frac{f(x)}{g(x)}\right) = ?$$

$$D\left(\frac{f(x)}{g(x)}\right) = \lim_{h \rightarrow 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{h g(x+h)g(x)} =$$

aggiungo e tolgo $f(x)g(x)$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x) - f(x)g(x+h) + f(x)g(x) - f(x)g(x)}{h g(x+h)g(x)} =$$

$$= \lim_{h \rightarrow 0} \frac{[f(x+h)g(x) - f(x)g(x)] + [f(x)g(x) - f(x)g(x+h)]}{h g(x+h)g(x)} =$$

$$= \lim_{h \rightarrow 0} \left[\frac{g(x) \overset{f'(x)}{\uparrow} (f(x+h) - f(x))}{h g(x+h)g(x)} - \frac{f(x) \overset{g'(x)}{\uparrow} (g(x+h) - g(x))}{h g(x+h)g(x)} \right]$$

$$= \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$D\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$