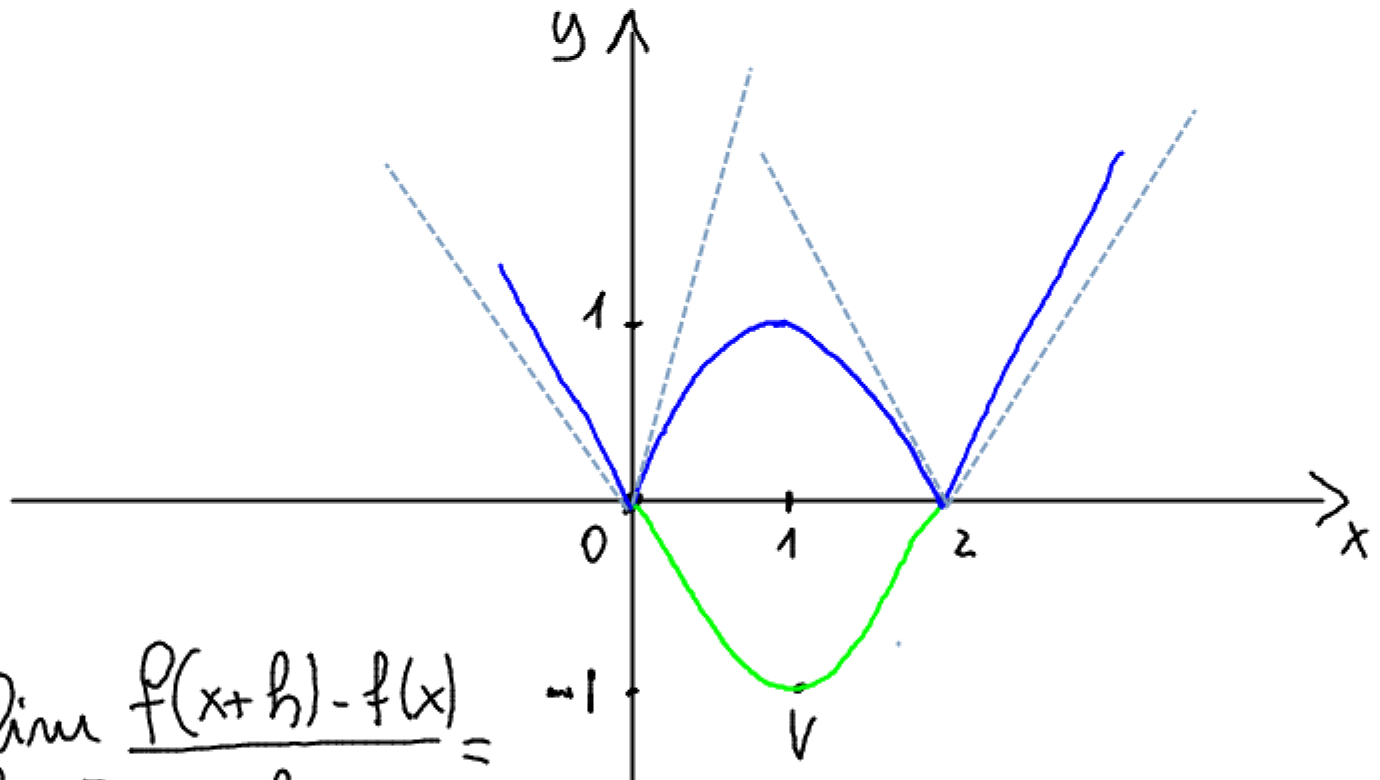
ESEMPIO

$f(x)=|x^2-2x|$ verificare se è derivabile.

$$f(x)=\begin{cases} x^2-2x & \text{per } x<0 \text{ o } x\geq 2 \\ -x^2+2x & \text{per } 0\leq x<2 \end{cases}$$

$x^2-2x\geq 0 \qquad x(x-2)\geq 0$

		0		2	x
x	$-$	$\bullet$	$+$		$+$
$x-2$	$-$		$-$	$\bullet$	$+$
$x(x-2)$	$+$	$\bullet$	$-$	$\bullet$	$+$



$$f'_-(0)=\lim_{h\to 0^-}\frac{f(x+h)-f(x)}{h}=$$

$$=\lim_{h\to 0^-}\frac{(x+h)^2-2(x+h)-\left[x^2-2x\right]}{h}=\lim_{h\to 0^-}\frac{\cancel{x^2}+h^2+2xh\cancel{-2x}-2h\cancel{-x^2}\cancel{+2x}}{h}=$$

$$=\lim_{h\to 0^-}\frac{\cancel{h}(h+2x-2)}{\cancel{h}}=2x-2\Big|_{x=0}=-2$$

$$f'_+(0)=\lim_{h\to 0^+}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0^+}\frac{-(x+h)^2+2(x+h)-\left[-x^2+2x\right]}{h}=$$

$$=\lim_{h\to 0^+}\frac{\cancel{-x^2}-h^2-2xh\cancel{+2x}+2h\cancel{+x^2}\cancel{-2x}}{h}=\lim_{h\to 0^+}\frac{-h^2-2xh+2h}{h}=$$

$$=\lim_{h\to 0^+}\frac{\cancel{h}(-h-2x+2)}{\cancel{h}}=(-2x+2)\Big|_{x=0}=2$$

$f(x)$ non è derivabile in $x=0$

$\boxed{\text{Calcolare } f'_-(2) \text{ e } f'_+(2) \text{ e verificare che } y=f(x) \text{ non è derivabile in } x=2}$

X CASA