

# FORMULE DI DUPLICAZIONE

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$$\text{sen } 2\alpha = \text{sen}(\alpha + \alpha) = \text{sen}\alpha \cos\alpha + \cos\alpha \text{sen}\alpha = 2\text{sen}\alpha \cos\alpha$$

$$\text{sen } 2\alpha = 2\text{sen}\alpha \cos\alpha$$

$$\cos 2\alpha = \cos(\alpha + \alpha) = \cos\alpha \cos\alpha - \text{sen}\alpha \text{sen}\alpha = \cos^2\alpha - \text{sen}^2\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \text{sen}^2\alpha$$

$$\text{tg } 2\alpha = \text{tg}(\alpha + \alpha) = \frac{\text{tg}\alpha + \text{tg}\alpha}{1 - \text{tg}\alpha \text{tg}\alpha} = \frac{2\text{tg}\alpha}{1 - \text{tg}^2\alpha}$$

$$\text{tg } 2\alpha = \frac{2\text{tg}\alpha}{1 - \text{tg}^2\alpha}$$

$$\text{ctg } 2\alpha = \text{ctg}(\alpha + \alpha) = \frac{\text{ctg}\alpha \text{ctg}\alpha - 1}{\text{ctg}\alpha + \text{ctg}\alpha} = \frac{\text{ctg}^2\alpha - 1}{2\text{ctg}\alpha}$$

$$\text{ctg } 2\alpha = \frac{\text{ctg}^2\alpha - 1}{2\text{ctg}\alpha}$$

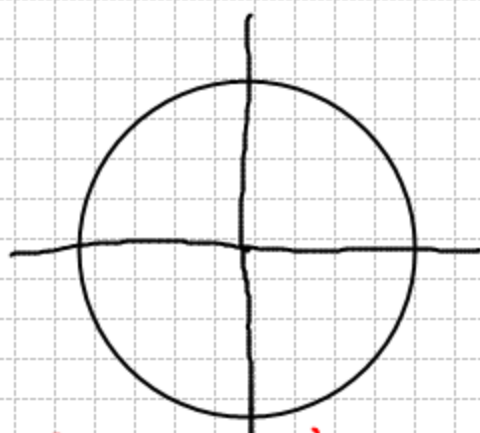
# FORMULE DI BISEZIONE

$$\operatorname{sen} \frac{\alpha}{2} = ? \quad \cos \frac{\alpha}{2} = ? \quad \operatorname{Tg} \frac{\alpha}{2} = ? \quad \operatorname{ctg} \frac{\alpha}{2} = ?$$

$$\begin{aligned} \cos \alpha &= \cos 2\left(\frac{\alpha}{2}\right) = \cos^2 \frac{\alpha}{2} - \operatorname{sen}^2 \frac{\alpha}{2} = \cos^2 \frac{\alpha}{2} - (1 - \cos^2 \frac{\alpha}{2}) \\ &= 2 \cos^2 \frac{\alpha}{2} - 1 \end{aligned}$$

$$\cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2}$$

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$



$$\cos \alpha = \cos 2\left(\frac{\alpha}{2}\right) = \left(\cos^2 \frac{\alpha}{2}\right) - \operatorname{sen}^2 \frac{\alpha}{2} = \left(1 - \operatorname{sen}^2 \frac{\alpha}{2}\right) - \operatorname{sen}^2 \frac{\alpha}{2} = 1 - 2 \operatorname{sen}^2 \frac{\alpha}{2}$$

$$\operatorname{sen}^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2}$$

$$\operatorname{sen} \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\operatorname{Tg} \frac{\alpha}{2} = \frac{\operatorname{sen} \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

$$\operatorname{Tg} \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\operatorname{sen} \alpha}$$

$$\operatorname{ctg} \frac{\alpha}{2} = \frac{\cos \frac{\alpha}{2}}{\operatorname{sen} \frac{\alpha}{2}} = \pm \sqrt{\frac{1 + \cos \alpha}{1 - \cos \alpha}}$$

$$\operatorname{ctg} \frac{\alpha}{2} = \frac{1 + \cos \alpha}{\operatorname{sen} \alpha}$$

(•) Dimostrazione

$$\operatorname{Tg} \frac{\alpha}{2} = \frac{\operatorname{sen} \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} = \frac{2 \operatorname{sen}^2 \frac{\alpha}{2}}{2 \operatorname{sen} \frac{\alpha}{2} \cos \frac{\alpha}{2}} = \frac{2 \operatorname{sen}^2 \frac{\alpha}{2}}{\operatorname{sen} \alpha} = \frac{1 - \cos \alpha}{\operatorname{sen} \alpha}$$

multiplico per  $\operatorname{sen} \frac{\alpha}{2}$

$$\operatorname{ctg} \frac{\alpha}{2} = \frac{\cos \frac{\alpha}{2}}{\operatorname{sen} \frac{\alpha}{2}} = \frac{2 \cos^2 \frac{\alpha}{2}}{2 \operatorname{sen} \frac{\alpha}{2} \cos \frac{\alpha}{2}} = \frac{1 + \cos \alpha}{\operatorname{sen} \alpha}$$

multiplico per  $\cos \frac{\alpha}{2}$