LIMITI NOTEVOLI

Forme indeterminate classiche

1) $\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1$ 2) $\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^x = e$

FIGLI

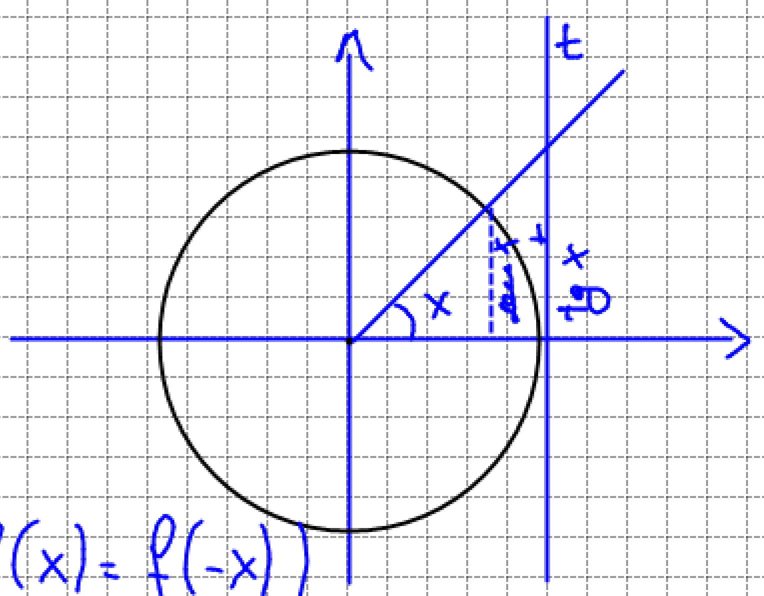
$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$; $\lim_{x \to -\infty} \left(1 + \frac{1}{x}\right)^x = e$

$\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$; $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

NIPOTI

$\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a$; $\lim_{x \to 0} \frac{\operatorname{Tg} x}{x} = 1$

Dimostrazione



1) $\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1$

$f(x) = \frac{\operatorname{sen} x}{x}$ PARI $(f(x) = f(-x))$

Quindi limitiamo il calcolo a $\lim_{x \to 0^+} \frac{\operatorname{sen} x}{x} = 1$

$$\operatorname{sen} x \leq x \leq \operatorname{Tg} x \quad \forall\, x \in \left[0; \frac{\pi}{2}\right)$$

divido per $\operatorname{sen} x \neq 0$ $\quad 1 \leq \frac{x}{\operatorname{sen} x} \leq \frac{1}{\cos x} \quad \forall x \in \left(0; \frac{\pi}{2}\right)$

$$\cos x \leq \frac{\operatorname{sen} x}{x} \leq 1$$

$$\underbrace{\lim_{x \to 0^+} \cos x}_{1} \leq \lim_{x \to 0^+} \frac{\operatorname{sen} x}{x} \leq \underbrace{\lim_{x \to 0^+} 1}_{1}$$

Per il teorema dei due carabinieri anche $\lim_{x \to 0^+} \frac{\operatorname{sen} x}{x} = 1$

Siccome $y = \frac{\operatorname{sen} x}{x}$ è PARI concludo che

$$\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1$$

2) $\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^x = e$ basta assegnare ad x valori grandi e vedere a quanto tende $\left(1 + \frac{1}{x}\right)^x$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \left[\frac{0}{0}\right] F.I.$$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2}\,\frac{(1+\cos x)}{(1+\cos x)} = \lim_{x\to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)} =$$

$$= \lim_{x\to 0} \frac{\operatorname{sen}^2 x}{x^2(1+\cos x)} = \lim_{x\to 0} \underbrace{\frac{\operatorname{sen} x}{x}}_{\to 1}\,\underbrace{\frac{\operatorname{sen} x}{x}}_{\to 1}\,\underbrace{\frac{1}{1+\cos x}}_{\to \frac{1}{2}} = \frac{1}{2}$$

- $$\lim_{x\to -\infty} \left(1+\frac{1}{x}\right)^x = \left[\left(1+\frac{1}{-\infty}\right)^{-\infty} \to (1+0^-)^{-\infty} \to \frac{1}{1^{\infty}}\right] F.I.$$

$$\lim_{x\to -\infty} \left(1+\frac{1}{x}\right)^x =$$ poniamo $y=-x$ quindi quando $x\to -\infty$ $y\to +\infty$.

$$= \lim_{y\to +\infty} \left(1-\frac{1}{y}\right)^{-y} = \lim_{y\to +\infty} \left(\frac{y-1}{y}\right)^{-y} = \lim_{y\to +\infty} \left(\frac{y}{y-1}\right)^{y} =$$

$$= \lim_{y\to +\infty} \left(1+\frac{1}{y-1}\right)^{y} = \lim_{y\to +\infty} \left(1+\frac{1}{y-1}\right)^{y-1}\left(1+\frac{1}{y-1}\right) =$$

poniamo $y-1=z$ se $y\to +\infty \Rightarrow z\to +\infty$.

$$= \lim_{z\to +\infty} \underbrace{\left(1+\frac{1}{z}\right)^{z}}_{z\to +\infty:\ \to e}\,\underbrace{\left(1+\frac{1}{z}\right)}_{z\to +\infty:\ \to 1} = e$$

- $$\lim_{x\to 0} \frac{\ln(1+x)}{x} =$$ poniamo $\frac{1}{x}=y$

$x\to 0 \Rightarrow y\to \infty$

$$= \lim_{x\to 0} \frac{1}{x}\ln(1+x) = \lim_{x\to 0} \ln(1+x)^{\frac{1}{x}} = \lim_{y\to \infty} \ln\left(1+\frac{1}{y}\right)^y =$$

$$= \ln e = 1.$$

$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$$

poniamo $y = e^x - 1$ $\quad x = \ln(y+1)$

$x \to 0 \Rightarrow \ln(1+y) \to 0 \Rightarrow$

$1+y \to 1 \Rightarrow y \to 0$

$$\lim_{x\to 0} \frac{e^x - 1}{x} = \lim_{y\to 0} \frac{y}{\ln(1+y)} = \lim_{y\to 0} \frac{1}{\frac{1}{y}\ln(1+y)} =$$

$$= \lim_{y\to 0} \frac{1}{\ln(1+y)^{\frac{1}{y}}} = 1$$

stesso discorso di prima!!

- $$\lim_{x\to \pi} \frac{\operatorname{sen} x}{x-\pi} = \left[\frac{0}{0}\right] \text{ F.I.}$$

$$\lim_{x\to \pi} \frac{\operatorname{sen} x}{x-\pi} =$$

poniamo $x - \pi = y$ $\quad$ se $x \to \pi$ $y \to 0$

$$= \lim_{y\to 0} \frac{\operatorname{sen}(y+\pi)}{y} = \lim_{y\to 0} \frac{-\operatorname{sen} y}{y} = -1$$

Per casa $\displaystyle \lim_{x\to 1} \frac{\log x}{e^x - e}$