$$\lim_{x\to\infty} f(x) = \ell$$

$\forall \varepsilon > 0 \ \exists\, I_\varepsilon(\ell)$ e con. $\exists\, I_M(\infty)$ /
$\forall x \in I_M(\infty)$ si ha:

$$|f(x) - \ell| < \varepsilon$$
$$|x| > M$$

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$$\lim_{x\to\infty} \frac{3x-8}{4x+x^2} = \lim_{x\to\infty} \frac{x\left(3-\frac{8}{x}\right)}{x^2\left(\frac{4}{x}+1\right)} = \lim_{x\to\infty} \frac{3}{x}$$

($3-\frac{8}{x} \to 0$, $\frac{4}{x} \to 0$)

$$= 0$$

$\forall \varepsilon > 0 \ \exists\, I_\varepsilon(0)$ e cons. $\exists\, I_M(\infty)$ /
$\forall x \in I_M$ si ha:

$$\begin{cases} \left|\frac{3x-8}{4x+x^2}\right| < \varepsilon \\ |x| > M \end{cases} \qquad \begin{cases} -\varepsilon < \frac{3x-8}{4x+x^2} < \varepsilon \\ x < -M \cup x > M \end{cases}$$

$$\textcircled{1} \begin{cases} \frac{3x-8}{4x+x^2} > -\varepsilon \\ \frac{3x-8}{4x+x^2} < \varepsilon \\ x < -M \cup x > M \end{cases}$$

$D_1 > 0 \quad 4x + x^2 > 0$

$x(4+x) > 0$

$x < -4 \cup x > 0$

$\textcircled{1} \quad \frac{3x-8}{4x+x^2} + \varepsilon > 0$

$$\frac{3x-8+4\varepsilon x+\varepsilon x^2}{4x+x^2} > 0$$

$N_1 > 0 \quad \varepsilon x^2 + x(4\varepsilon+3) - 8 > 0$

$$x_{1/2} = \frac{-(4\varepsilon+3) \pm \sqrt{16\varepsilon^2+24\varepsilon+9+32\varepsilon}}{2\varepsilon} = \frac{-(4\varepsilon+3) \pm \sqrt{16\varepsilon^2+56\varepsilon+9}}{2\varepsilon}$$

$$x_1 = \frac{-(4\varepsilon+3) - \sqrt{16\varepsilon^2+56\varepsilon+9}}{2\varepsilon}$$

$$x_2 = \frac{-(4\varepsilon+3) + \sqrt{16\varepsilon^2+56\varepsilon+9}}{2\varepsilon}$$

$x < x_1 \cup x > x_2$

$\varepsilon = 0{,}1$

$$\frac{-3{,}4 - \sqrt{0{,}16+5.6+9}}{0{,}2} = -36$$

$$= 2$$

	x_1	-4		0		x_2	
N_1	+	0	−	‖	−	0	+
D_1	+	‖	+	0	−	0	+
N_1/D_1	+	0	−	∄	+	0	+
	sì		no		no		sì

$S_1: x < x_1 \cup -4 < x < 0 \cup x > x_2$

② $\frac{3x-8}{4x+x^2} < \varepsilon$

$\frac{3x-8}{4x+x^2} - \varepsilon < 0$

$\frac{3x-8-4\varepsilon x-\varepsilon x^2}{4x+x^2} < 0$

$N_2 > 0 \quad -\varepsilon x^2 + x(3-4\varepsilon) - 8 > 0$

$x_{3/4} = \frac{-3+4\varepsilon \mp \sqrt{9+16\varepsilon^2-24\varepsilon-32\varepsilon}}{-2\varepsilon} = \frac{3-4\varepsilon \mp \sqrt{16\varepsilon^2-56\varepsilon+9}}{2\varepsilon}$

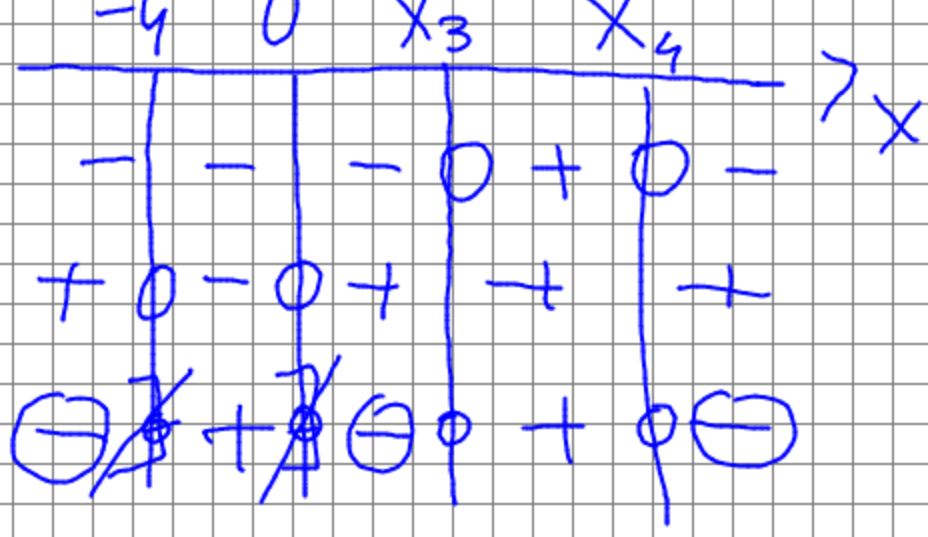
$= \frac{2{,}6 \mp 1{,}9}{0{,}2}$ $\quad (-)\, x_3 = 3{,}5 \quad (+)\, x_4 = 21$

$x_3 < x < x_4$

$D_2 > 0 \quad 4x + x^2 > 0$

$x(4+x) > 0$

$x < -4 \cup x > 0$



S_2 : $x < -4 \cup 0 < x < x_3 \cup x > x_4$

$x < -M \quad \cup \quad x > M$

$$\lim_{x \to \infty} f(x) = \infty$$

$\forall M > 0 \; \exists \, I_M(\infty) \in \text{CORR.} \; \nexists \, I_\sigma(\infty) /$
$\forall x \in I_\sigma(\infty)$ SI HA:

$$|f(x)| > M$$