

# FUNZIONI RAZIONALI FRATTE (LIMITI $x \rightarrow \infty$ )

$$\lim_{x \rightarrow \infty} \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \dots + b_1 x + b_0} =$$

$$g(N) = n \quad g(D) = m$$

$$= \lim_{x \rightarrow \infty} \frac{x^n \left( a_n + \frac{a_{n-1}}{x} + \dots + \frac{a_1}{x^{n-1}} + \frac{a_0}{x^n} \right)}{x^m \left( b_m + \frac{b_{m-1}}{x} + \dots + \frac{b_1}{x^{m-1}} + \frac{b_0}{x^m} \right)} =$$

$$= \begin{cases} n = m & = \frac{a_n}{b_m} \\ n > m & = \infty \\ n < m & = \frac{1}{\infty} \rightarrow 0 \end{cases}$$

## ESEMPIO

$$\lim_{x \rightarrow \infty} \frac{4x^5 + 5}{3x^4 + 2x + 1} = \lim_{x \rightarrow \infty} \frac{x^4 \left( 4 + \frac{5}{x^4} \right)}{x^4 \left( 3 + \frac{2}{x^3} + \frac{1}{x^4} \right)} = \frac{4}{3}$$

$y = \frac{4}{3}$  Asintoto orizzontale.

## ESEMPIO

$$\lim_{x \rightarrow \infty} \frac{3x^3 + 2x}{4x} = \lim_{x \rightarrow \infty} \frac{x^2 \left( 3 + \frac{2}{x^2} \right)}{4x} = \infty$$

## ESEMPIO

$$\lim_{x \rightarrow \infty} \frac{5}{4x^2 + 3} = 0$$

# LIMITI FONDAMENTALI

LIMITE INFINITO  
ALL'INFINITO ( $m \in \mathbb{N}_0$ )

$$\lim_{x \rightarrow +\infty} x^{2m} = +\infty$$

$$\lim_{x \rightarrow +\infty} x^{2m+1} = +\infty$$

$$\lim_{x \rightarrow -\infty} x^{2m+1} = -\infty$$

$$\lim_{x \rightarrow +\infty} e^x = +\infty$$

$$\lim_{x \rightarrow +\infty} \ln x = +\infty$$

$$\lim_{x \rightarrow +\infty} \log_a x = -\infty \quad (0 < a < 1)$$

$$\lim_{x \rightarrow +\infty} \log_a x = +\infty \quad (a > 1)$$

$$\lim_{x \rightarrow \pm\infty} \cosh x = +\infty$$

$$\lim_{x \rightarrow +\infty} \sinh x = +\infty$$

$$\lim_{x \rightarrow -\infty} \sinh x = -\infty$$

LIMITE FINITO  
ALL'INFINITO ( $m \in \mathbb{N}_0$ )

$$\lim_{x \rightarrow \pm\infty} \frac{1}{x^n} = 0$$

$$\lim_{x \rightarrow +\infty} \frac{1}{\sqrt[2m]{x}} = 0$$

$$\lim_{x \rightarrow \pm\infty} \frac{1}{\sqrt[2m+1]{x}} = 0$$

$$\lim_{x \rightarrow -\infty} e^x = 0$$

$$\lim_{x \rightarrow -\infty} a^x = 0 \quad (a > 1)$$

$$\lim_{x \rightarrow +\infty} a^x = 0 \quad (0 < a < 1)$$

$$\lim_{x \rightarrow +\infty} \operatorname{arctg} x = \frac{\pi}{2}$$

$$\lim_{x \rightarrow -\infty} \operatorname{arctg} x = -\frac{\pi}{2}$$

$$\lim_{x \rightarrow +\infty} \operatorname{tgh} x = 1$$

$$\lim_{x \rightarrow -\infty} \operatorname{tgh} x = -1$$

LIMITE INFINITO  
IN UN PUNTO ( $m \in \mathbb{N}_0$ )

$$\lim_{x \rightarrow 0} \frac{1}{x^{2m}} = +\infty$$

$$\lim_{x \rightarrow 0^+} \frac{1}{x^{2m+1}} = +\infty$$

$$\lim_{x \rightarrow 0^-} \frac{1}{x^{2m+1}} = -\infty$$

$$\lim_{x \rightarrow 0^+} \ln x = -\infty$$

$$\lim_{x \rightarrow \frac{\pi}{2}^-} \operatorname{tg} x = +\infty$$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} \operatorname{tg} x = -\infty$$

$$\lim_{x \rightarrow 0^+} \operatorname{ctg} x = +\infty$$

$$\lim_{x \rightarrow 0^-} \operatorname{ctg} x = -\infty$$

$$\lim_{x \rightarrow 0^+} \operatorname{dgh} x = +\infty$$

$$\lim_{x \rightarrow 0^-} \operatorname{dgh} x = -\infty$$