

$$\ln^4(1-3x) - 13\ln^2(1-3x) + 36 \leq 0$$

C.E. : $1-3x > 0 \Rightarrow -3x > -1$

$$\ln^2(1-3x) = t \quad x < \frac{1}{3}$$

$$t^2 - 13t + 36 \leq 0$$

$$t = \frac{13 \pm \sqrt{169 - 144}}{2} = \frac{13 \pm 5}{2} = \begin{matrix} 9 \\ 4 \end{matrix}$$

$$\begin{matrix} 4 & 9 \\ + & | \ominus | & + \\ \hline & t & \end{matrix} \quad 4 \leq t \leq 9$$

$$\begin{cases} \ln^2(1-3x) \geq 4 \quad (\star) \\ \ln^2(1-3x) \leq 9 \quad (\circ) \\ x \leq \frac{1}{3} \end{cases} \begin{cases} x \leq \frac{1-e^2}{3} \cup x \geq \frac{1-e^{-2}}{3} \\ \frac{1-e^3}{3} \leq x \leq \frac{1-e^{-3}}{3} \\ x \leq \frac{1}{3} \end{cases}$$

(\star) $\ln^2(1-3x) \geq 4$

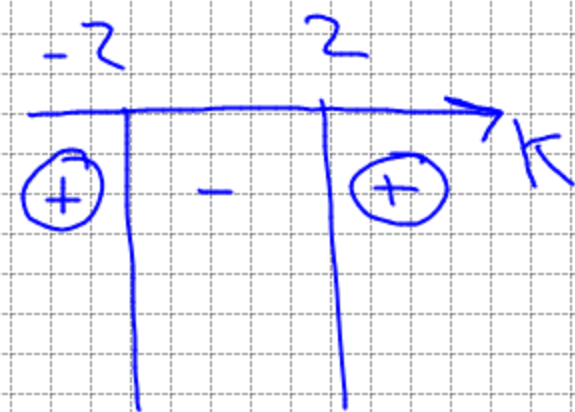
$$\ln(1-3x) = k \Rightarrow k^2 \geq 4 \quad k^2 - 4 \geq 0$$

$$k \leq -2 \cup k \geq 2$$

$$\ln(1-3x) \leq -2 \cup \ln(1-3x) \geq 2$$

$$1-3x \leq e^{-2} \cup 1-3x \geq e^2$$

$$x \geq \frac{1-e^{-2}}{3} \cup x \leq \frac{1-e^2}{3}$$



(\circ) $\ln^2(1-3x) \leq 9$

$$\ln(1-3x) = k \Rightarrow k^2 - 9 \leq 0$$

$$k^2 = 9 \quad -3 \leq k \leq 3$$

$$\begin{cases} \ln(1-3x) \geq -3 \\ \ln(1-3x) \leq 3 \end{cases} \begin{cases} k = \pm 3 \\ 1-3x \geq e^{-3} \\ 1-3x \leq e^3 \end{cases}$$

$$\begin{cases} x \leq \frac{1-e^{-3}}{3} \\ x \geq \frac{1-e^3}{3} \end{cases}$$

