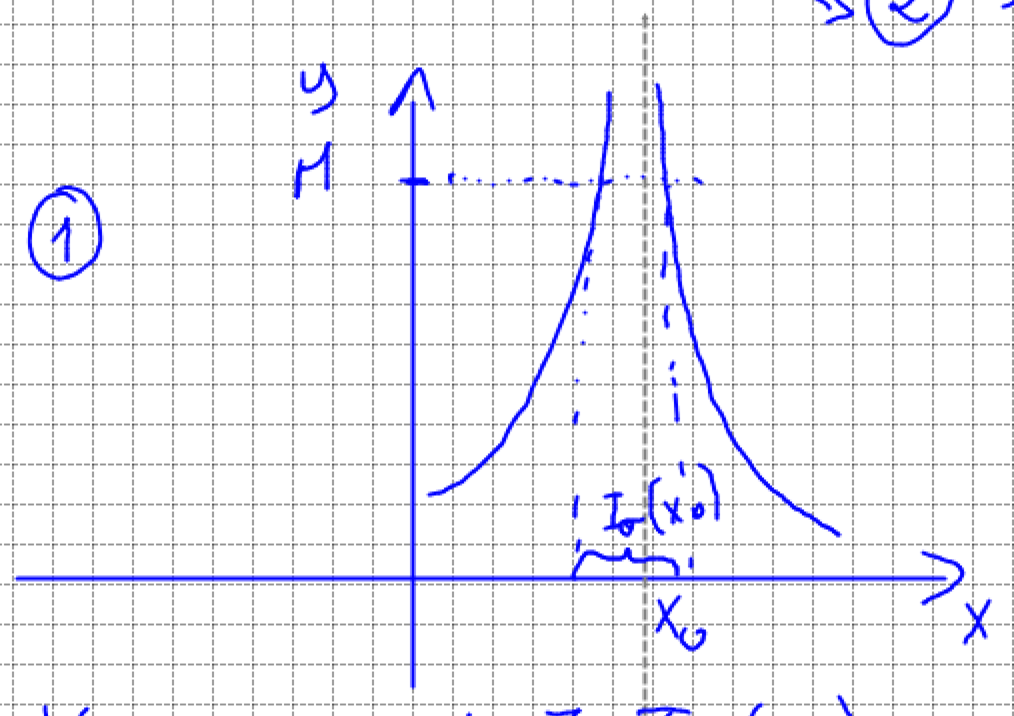
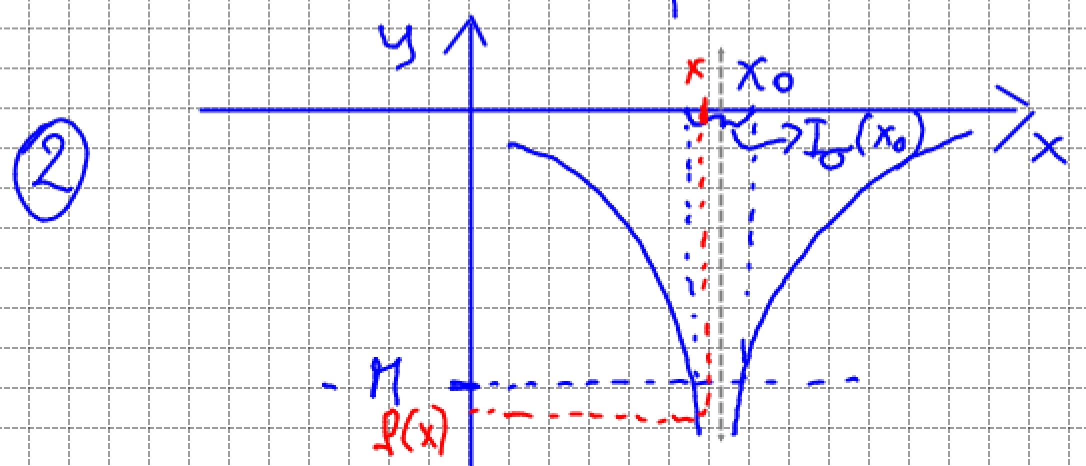
LIMITE INFINITO-FINITO

$$\lim_{x \to x_0} f(x) = \infty \quad \begin{cases} (1) \ \lim_{x \to x_0} f(x) = +\infty \\ (2) \ \lim_{x \to x_0} f(x) = -\infty \end{cases}$$



$\forall M > 0$ "grande" $\exists\, I_M(+\infty)$ e corrispondentemente $\exists\, I_\sigma(x_0)$ / $\forall x \in I_\sigma(x_0)$ si ha che $f(x) \in I_M(+\infty)$ cioè:

$$f(x) > M$$



$\forall -M < 0$ "grande" $\exists\, I_{-M}(-\infty)$ e corrispondentemente $\exists\, I_\sigma(x_0)$ / $\forall x \in I_\sigma(x_0)$ si ha che $f(x) \in I_{-M}(-\infty)$ cioè:

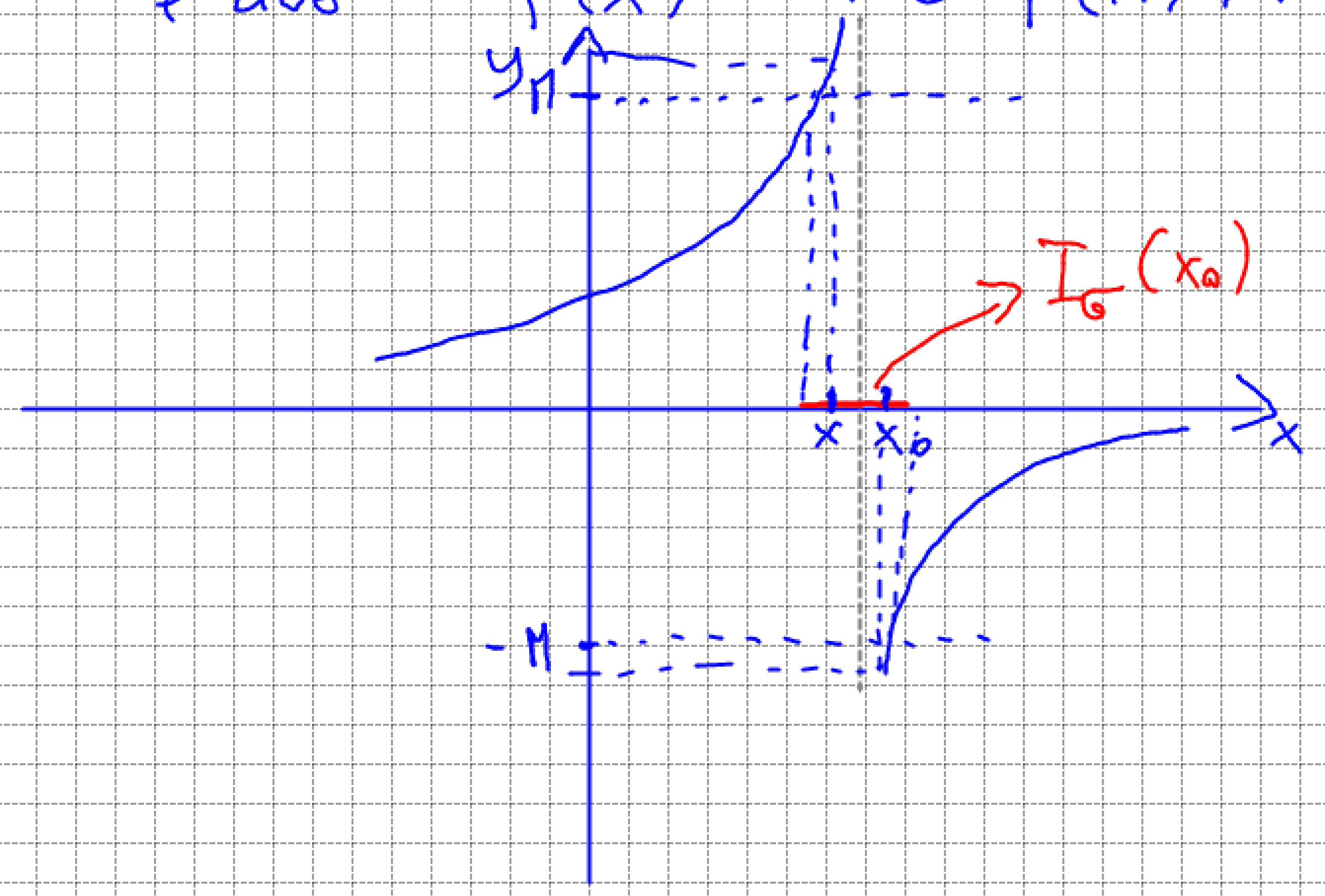
$$f(x) < -M$$

$$\boxed{\lim_{x \to x_0} f(x) = \infty}$$

$\forall M > 0 \ \exists\, I(\infty) = I_{-M}(-\infty) \cup I_M(+\infty)$ e corrispondentemente $\exists\, I_\sigma(x_0)$ / $\forall x \in I_\sigma(x_0)$ si ha $f(x) \in I(\infty)$ ovvero

$$|f(x)| > M$$

e cioè $\quad f(x) < -M \cup f(x) > M$



LIMITE SINISTRO

$$\boxed{\lim_{x \to x_0^-} f(x) = +\infty}$$

$\forall M > 0 \ \exists I_M(+\infty)$ e corrisp. $\exists I_\sigma^-(x_0) \mid \forall x \in I_\sigma^-(x_0)$ si ha $f(x) > M$

$$\boxed{\lim_{x \to x_0^-} f(x) = -\infty}$$

$\forall -M < 0 \ \exists I_{-M}(-\infty)$ e corrisp. $\exists I_\sigma^-(x_0) \mid \forall x \in I_\sigma^-(x_0)$ si ha $f(x) < -M$

LIMITE DESTRO

$$\boxed{\lim_{x \to x_0^+} f(x) = +\infty}$$

$\forall M > 0 \ \exists I_M(+\infty)$ e corrisp. $\exists I_\sigma^+(x_0) \mid \forall x \in I_\sigma^+(x_0)$ si ha $f(x) > M$

$$\boxed{\lim_{x \to x_0^+} f(x) = -\infty}$$

$\forall -M < 0 \ \exists I_{-M}(-\infty)$ e corrisp. $\exists I_\sigma^+(x_0) \mid \forall x \in I_\sigma^+(x_0)$ si ha che $f(x) < -M$

ESEMPIO

Verificare il seguente limite

$$\lim_{x \to 0^+} \left[\frac{1}{x} + \frac{3}{x^2} \right] = +\infty$$

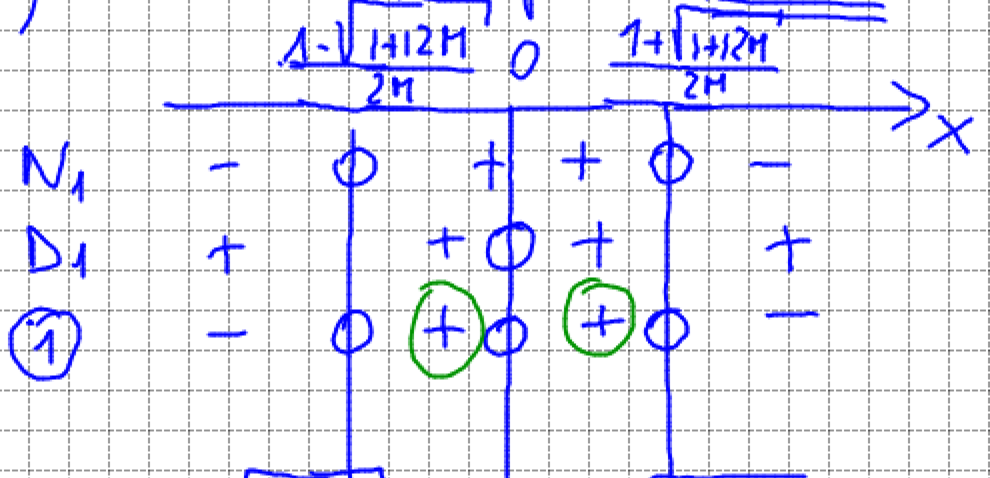
$\forall M > 0 \; \exists \; I_M(+\infty)$ e corrisp. $\exists \; I_\sigma^+(0) \; / \; \forall x \in I_\sigma^+(0)$ si ha $f(x) > M$ ovvero

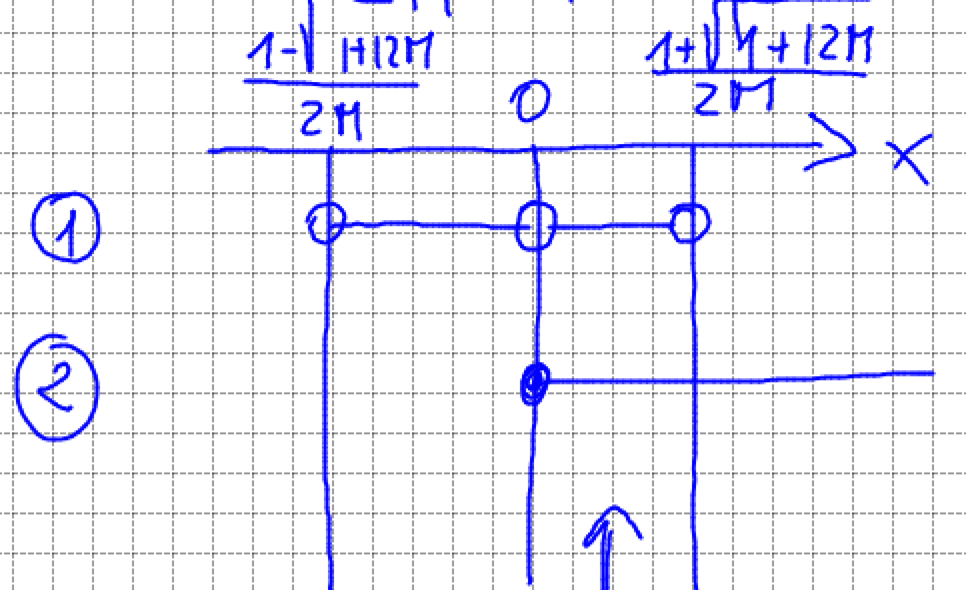
$$\begin{cases} \frac{1}{x} + \frac{3}{x^2} > M & (1) \\ x \geq 0 & (2) \end{cases} \qquad \begin{cases} \frac{x + 3 - Mx^2}{x^2} > 0 \\ x \geq 0 \end{cases}$$

(1) $N_1 > 0 \quad -Mx^2 + x + 3 > 0 \qquad x_{1,2} = \frac{-1 \mp \sqrt{1+12M}}{-2M} = \frac{1 \mp \sqrt{1+12M}}{2M}$

$$\frac{1 - \sqrt{1+12M}}{2M} < x < \frac{1 + \sqrt{12M+1}}{2M}$$

$D_1 > 0 \quad x^2 > 0$ sempre + escluso $x = 0$





$$x \in \left[0 ; \frac{1 + \sqrt{1+12M}}{2M} \right) = I_\sigma^+(0)$$