

PROPRIETÀ LOGARITMI

$b_1 > 0$ e $b_2 > 0$:

1) LOGARITMO DEL PRODOTTO

$$\log_a (b_1 \cdot b_2) = \log_a b_1 + \log_a b_2$$

Dim

$$x_1 = \log_a b_1 \quad x_2 = \log_a b_2$$

$$a^{x_1} = b_1 \quad a^{x_2} = b_2$$

$$b_1 \cdot b_2 = a^{x_1} \cdot a^{x_2}$$

$$b_1 \cdot b_2 = a^{x_1 + x_2}$$

$$x_1 + x_2 = \log_a (b_1 \cdot b_2)$$

$$\log_a b_1 + \log_a b_2 = \log_a (b_1 \cdot b_2)$$

2) LOGARITMO DEL QUOZIENTE

$$\log_a \left(\frac{b_1}{b_2} \right) = \log_a b_1 - \log_a b_2$$

Dim

$$x_1 = \log_a b_1 \quad x_2 = \log_a b_2$$

$$b_1 = a^{x_1} \quad b_2 = a^{x_2}$$

$$\frac{b_1}{b_2} = \frac{a^{x_1}}{a^{x_2}}$$

$$\frac{b_1}{b_2} = a^{x_1 - x_2}$$

$$x_1 - x_2 = \log_a \left(\frac{b_1}{b_2} \right)$$

$$\log_a b_1 - \log_a b_2 = \log_a \left(\frac{b_1}{b_2} \right)$$

3) LOGARITMO DI UNA POTENZA

Se $b > 0$

$$\log_a (b)^k = k \log_a b$$

Dim

poniamo
 $x = \log_a b \Rightarrow a^x = b$ eleviamo
ambo i membri alla k : $a^{kx} = b^k$

$$\Rightarrow kx = \log_a b^k \Rightarrow k \log_a b = \log_a b^k$$

$$\log_{10} 100 = ?$$

$$10^? = 100$$

$$10^? = 10^2$$

$$? = 2$$

$$\log_{10} 10^2 = ?$$

$$2 \log_{10} 10 = 2 \cdot 1$$

$$\log_2 64 = \log_2 2^6 =$$

$$= 6$$

$$\log_2 ? = ? \quad 2^? = 2$$

$$? = 1$$

$$4) \log_a 1 = 0, \log_a a = 1$$

5) CAMBIAMENTO DI BASE

2/2

$$\log_a b = \frac{\log_c b}{\log_c a}$$

Dim

Poniamo

$$x = \log_a b$$

$$y = \log_c a$$

$$a^x = b$$

$$c^y = a$$

$$(c^y)^x = b \Leftrightarrow c^{xy} = b$$

$$xy = \log_c b$$

$$\log_a b \cdot \log_c a = \log_c b$$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

OSSERVA!!

→ cambia la base in a

$$\log_{\frac{1}{a}} b = \frac{\log_a b}{\log_a \frac{1}{a}} = \frac{\log_a b}{\log_a (a)^{-1}} = \frac{\log_a b}{-1} = -\log_a b$$

$$\log_{\frac{1}{a}} b = -\log_a b$$