ESPONENZIALI

$y = x$ $\quad y = x^2$ $\quad y = x^n$ POTENZE

$$\boxed{y = a^x}$$

funzione esponenziale (fissate alcune condizioni su a)

REGOLE POTENZE

1. $a^0 = 1$
2. $a^1 = a$, $a^2 = a \cdot a$, ..., $a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n\text{-volte}}$
3. $a^{-n} = \dfrac{1}{a^n}$
4. $a^{n+m} = a^n \cdot a^m$
5. $(a^m)^n = a^{m \cdot n}$
6. $a^{m-n} = a^m \cdot \dfrac{1}{a^n} = \dfrac{a^m}{a^n}$
7. $a^{\frac{1}{n}} = \sqrt[n]{a}$
8. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

FUNZIONE $y = a^x$

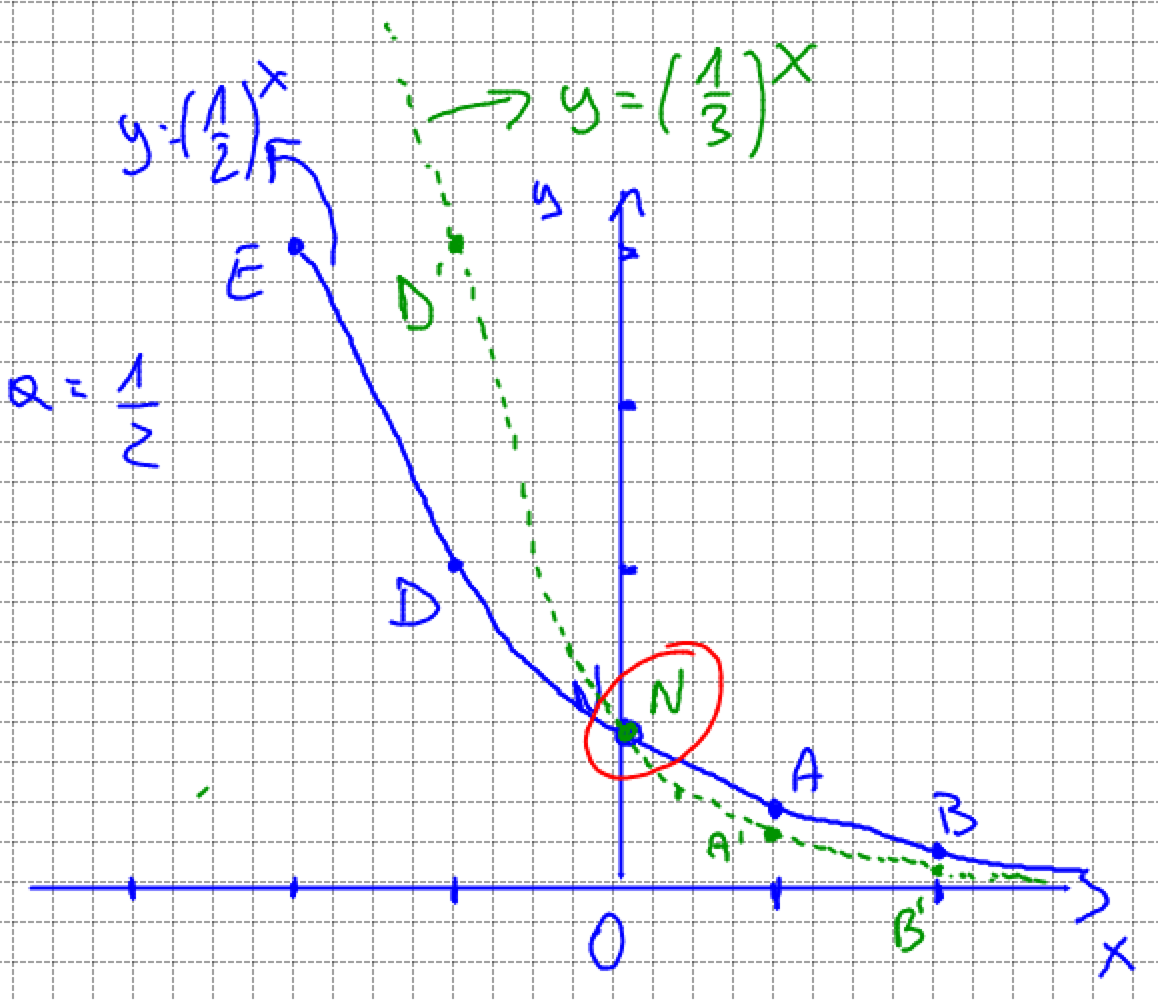
$y = a^x$

1. $0 < a < 1$
2. $a > 1$

1. $\boxed{0 < a < 1}$ supponiamo $a = \dfrac{1}{2}$

$y = \left(\dfrac{1}{2}\right)^x$

	x	y
N	0	1
A	1	1/2
B	2	1/4
C	3	1/8
D	-1	2
E	-2	4
F	-3	8

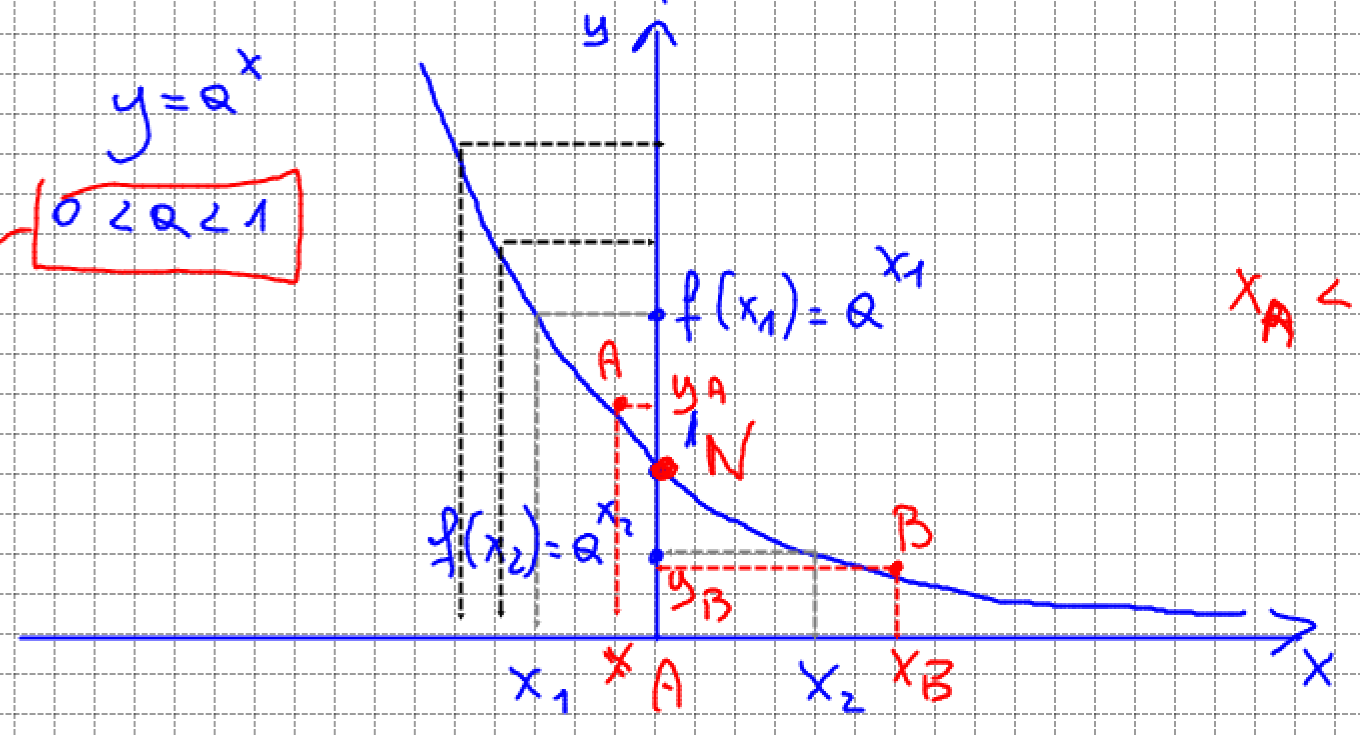


$y = \left(\dfrac{1}{3}\right)^x$

	x	y
N	0	1
A'	1	1/3
B'	2	1/9
C'	-1	3
D'	-2	9

$y = a^x$

$\boxed{0 < a < 1}$



$x_A < x_B \Rightarrow y_A > y_B$

$\Downarrow$

$a^{x_A} > a^{x_B}$

- $y > 0$
- $y = a^x$ passano tutte per $(0;1)$
- $y = a^x$ è decrescente: $\forall\, x_1, x_2 \in \mathbb{R}$ con $\boxed{x_1 < x_2 \Leftrightarrow a^{x_1} > a^{x_2}}$ (*)

ESEMPIO

$$\left(\frac{1}{3}\right)^{x+1} < \left(\frac{1}{3}\right)^{3x} \overset{(\bullet)}{\iff} x+1 > 3x \qquad 3x - x < 1$$

$$2x < 1 \qquad x < \frac{1}{2}$$

$0<a<1 \qquad a^{x_1} < a^{x_2} \iff x_1 > x_2$

$$y = \left(\frac{1}{3}\right)^{x+1} \qquad y = \left(\frac{1}{3}\right)^{x} \cdot \left(\frac{1}{3}\right)^{1} \qquad y = \frac{1}{3}\left(\frac{1}{3}\right)^{x}$$

$$y = \frac{1}{3} \cdot \frac{1}{3^x} \qquad \text{se } x \to +\infty \quad \frac{1}{3^x} \to 0$$

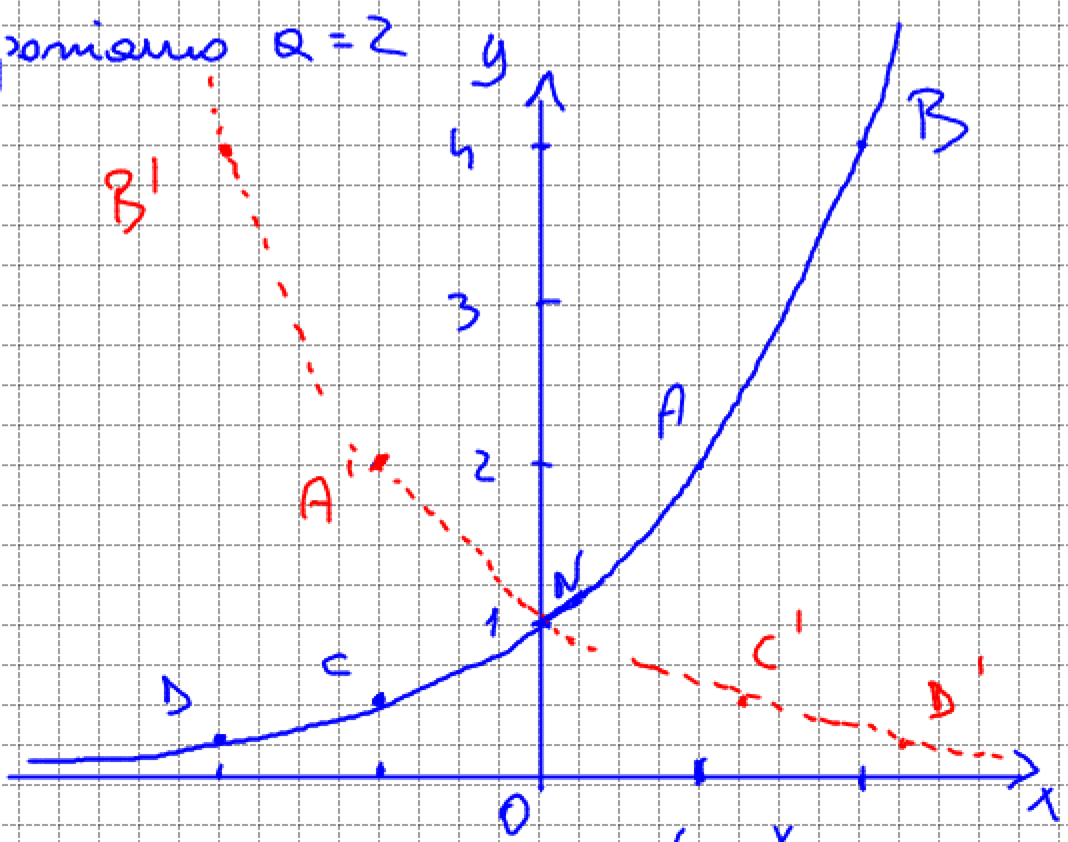
2. $\boxed{a > 1}$ $\quad y = a^x$ supponiamo $a = 2$

$y = 2^x$

	x	y
N	0	1
A	1	2
B	2	4
C	-1	$\frac{1}{2}$
D	-2	$\frac{1}{4}$

$$2^{-1} = \frac{1}{2^1} = \left(\frac{1}{2}\right)^1$$

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$



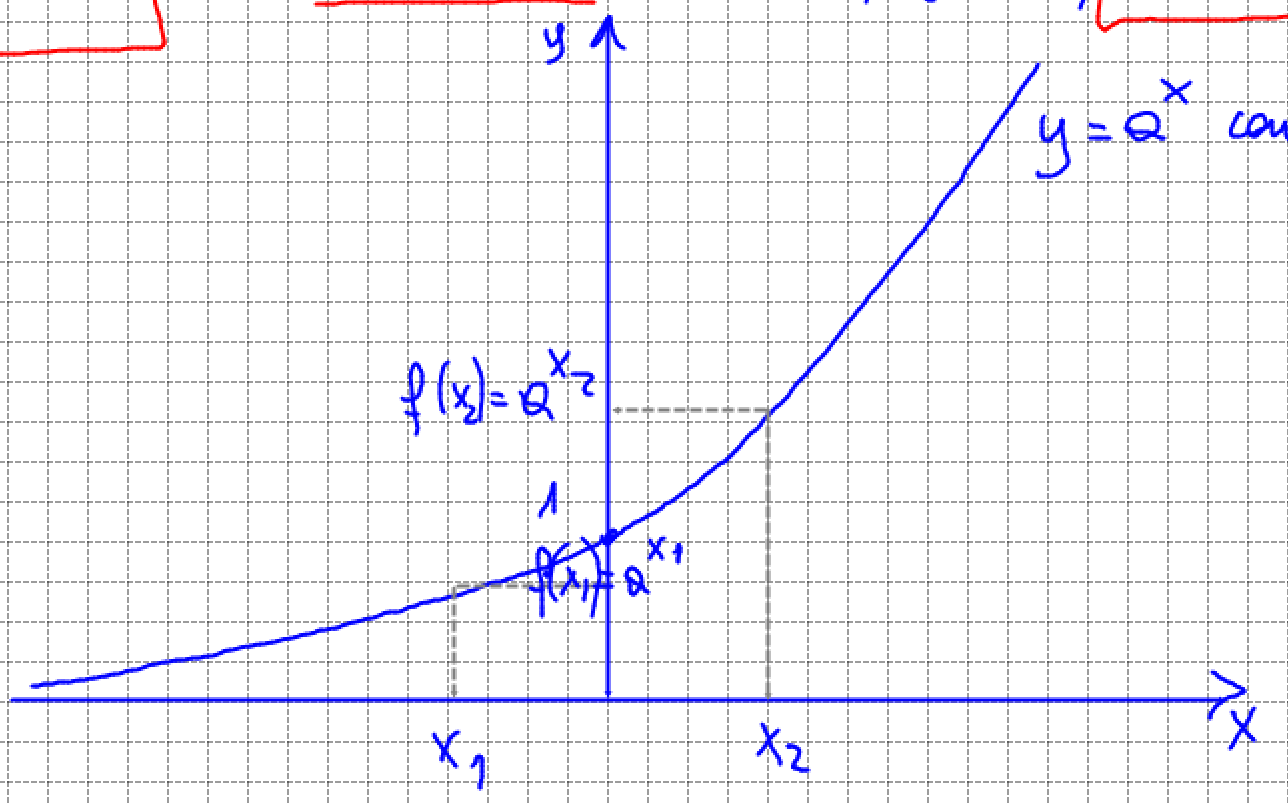
Questa curva è la curva simmetrica di $y = \left(\frac{1}{2}\right)^x$ rispetto all'asse y $(x = 0)$

SIMMETRIA $S \begin{cases} y = y' \\ x = -x' \end{cases}$ $\qquad y = \left(\frac{1}{2}\right)^x \xrightarrow{S} y' = \left(\frac{1}{2}\right)^{-x'}$

$$y' = 2^{x'}$$

PROPRIETÀ $(a > 1)$

- $y > 0$
- $y = a^x$ passano tutte per $(0;1)$
- $\boxed{y = a^x \quad a > 1}$ è crescente se $\forall x_1, x_2 \in \mathbb{R}$, $\boxed{x_1 < x_2 \Rightarrow a^{x_1} < a^{x_2}}$



ESEMPIO

$e = 2{,}718281\ldots\ldots$

$$e^x - e^{3x-3} < 0$$

$$e^x < e^{3x-3} \iff x < 3x - 3 \qquad 3x - x > 3 \quad 2x > 3 \quad x > \frac{3}{2}$$