

PRODOTTO

Dati due numeri complessi $z_1 = a_1 + ib_1$, $z_2 = a_2 + ib_2$

$$\begin{aligned} z_1 z_2 &= (a_1 + ib_1)(a_2 + ib_2) = a_1 a_2 + ia_1 b_2 + ia_2 b_1 + i^2 b_1 b_2 = \\ &= \underbrace{(a_1 a_2 - b_1 b_2)}_{\text{Re}(z_1 z_2)} + i \underbrace{(a_1 b_2 + a_2 b_1)}_{\text{Im}(z_1 z_2)} \end{aligned}$$

ESEMPIO

$$z = a + ib \quad \bar{z} = a - ib$$

$$z \bar{z} = (a + ib)(a - ib) = a^2 - a^2 i + a^2 i - \overset{-1}{i^2} b^2 = a^2 + b^2$$

$$\|z\| = a^2 + b^2 \quad (\text{NORMA})$$

POTENZA M-ESIMA DI UN NUMERO COMPLESSO

$z = a + ib$ numero complesso.

$$z^0 = (a + ib)^0 = 1$$

$$z^1 = (a + ib)^1 = a + ib$$

$$\begin{aligned} z^2 &= (a + ib)^2 = (a + ib)(a + ib) = a^2 + a^2 i + a^2 i + i^2 b^2 = \\ &= a^2 + 2abi - b^2 = \underbrace{(a^2 - b^2)}_{\text{Re}(z^2)} + i \underbrace{(2ab)}_{\text{Im}(z^2)} \end{aligned}$$

$$z^m = (a + ib)^m = \underbrace{(a + ib) \cdot (a + ib) \cdot \dots \cdot (a + ib)}_{m\text{-volte}}$$

$$i^0 = 1$$

$$i^1 = i$$

$$i^2 = -1$$

$$i^3 = i^2 \cdot i = -1(i) = -i$$

$$i^4 = i^3 \cdot i = (-i)(i) = -(-1) = 1$$

$$i^5 = i^4 \cdot i = 1(i) = i$$

$$i^6 = i^5 \cdot i = i(i) = -1$$

si ripetono.

REGOLA

$$i^{4m} = i^0 = 1$$

$$i^{4m+1} = i^1 = i$$

$$i^{4m+2} = i^2 = -1$$

$$i^{4m+3} = i^3 = -i$$

DIVISIONE

$$z_1 = a_1 + ib_1, \quad z_2 = a_2 + ib_2$$

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{a_1 + ib_1}{a_2 + ib_2} = \frac{(a_1 + ib_1)(a_2 - ib_2)}{(a_2 + ib_2)(a_2 - ib_2)} = \frac{(a_1 a_2 + b_1 b_2) + i(a_2 b_1 - a_1 b_2)}{a_2^2 + b_2^2} \\ &= \left[\frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} \right] + i \left[\frac{a_2 b_1 - a_1 b_2}{a_2^2 + b_2^2} \right] \end{aligned}$$