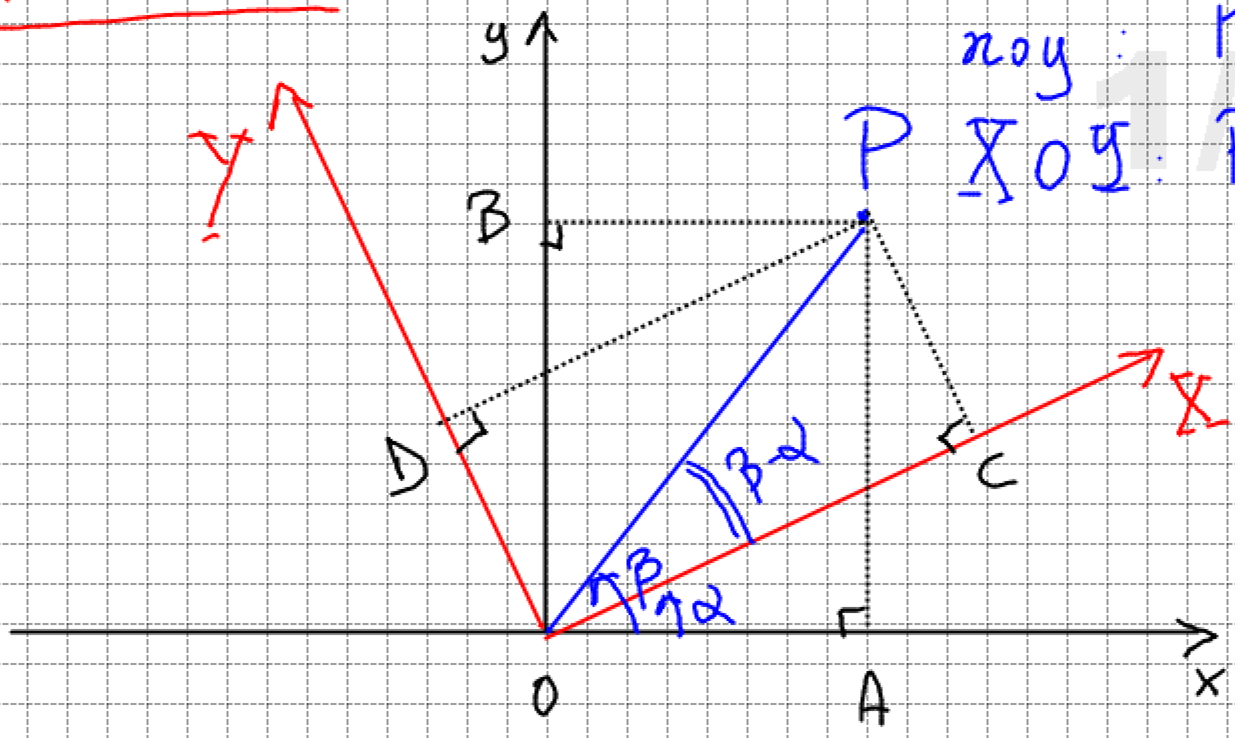
ROTAZIONE

$xOy: P(x;y)$

$XOY: P(X;Y)$



$x = \overline{OA} = \overline{BP} = \overline{OP}\cos\beta \qquad y = \overline{OB} = \overline{AP} = \overline{OP}\,\text{sen}\,\beta$

$X = \overline{OC} = \overline{DP} = \overline{OP}\cos(\beta-\alpha) \qquad Y = \overline{OD} = \overline{CP} = \overline{OP}\,\text{sen}(\beta-\alpha)$

$$X = \overline{OP}\left(\cos\beta\cos\alpha + \text{sen}\,\beta\,\text{sen}\,\alpha\right) = \left(\overline{OP}\cos\beta\right)\cos\alpha + \left(\overline{OP}\,\text{sen}\,\beta\right)\text{sen}\,\alpha$$

$$\boxed{X = x\cos\alpha + y\,\text{sen}\,\alpha}$$

$$Y = \overline{OP}\left(\text{sen}\,\beta\cos\alpha - \cos\beta\,\text{sen}\,\alpha\right) = \left(\overline{OP}\,\text{sen}\,\beta\right)\cos\alpha - \left(\overline{OP}\cos\beta\right)\text{sen}\,\alpha$$

$$\boxed{Y = y\cos\alpha - x\,\text{sen}\,\alpha}$$

$$R: \begin{cases} X = x\cos\alpha + y\,\text{sen}\,\alpha \\ Y = -x\,\text{sen}\,\alpha + y\cos\alpha \end{cases}$$

$$R': \begin{cases} x = X\cos\alpha - Y\,\text{sen}\,\alpha \\ y = X\,\text{sen}\,\alpha + Y\cos\alpha \end{cases}$$

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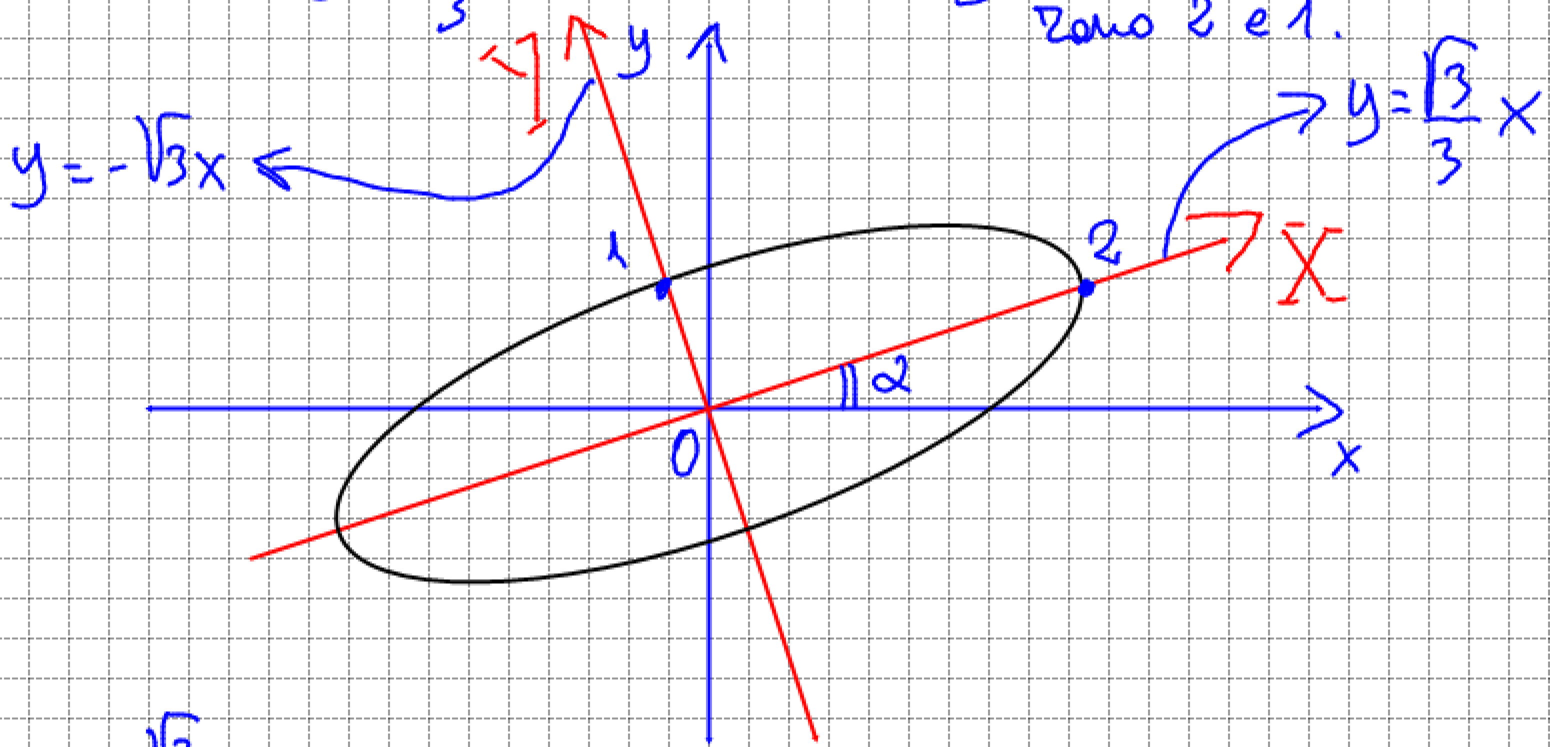
Dato l'ellisse i cui assi di simmetria sono:

$\underline{X}$: $y=\frac{\sqrt{3}}{3}x$ $\quad$ $\underline{Y}$: $y=-\sqrt{3}x$ e i semiassi misure: sono 2 e 1.



$$\frac{\sqrt{3}}{3}=\operatorname{Tg}\alpha \qquad \alpha=30^\circ$$

$$\frac{\underline{X}^2}{4}+\frac{\underline{Y}^2}{1}=1$$

$$\begin{cases}\underline{X}=x\cos 30^\circ+y\,\operatorname{sen}30^\circ\\ \underline{Y}=-x\,\operatorname{sen}30^\circ+y\cos 30^\circ\end{cases} \Rightarrow \begin{cases}\underline{X}=\frac{\sqrt{3}}{2}x+\frac{1}{2}y\\ \underline{Y}=-\frac{1}{2}x+\frac{\sqrt{3}}{2}y\end{cases}$$

$$\frac{\left(\frac{\sqrt{3}}{2}x+\frac{1}{2}y\right)^2}{4}+\frac{\left(-\frac{1}{2}x+\frac{\sqrt{3}}{2}y\right)^2}{1}=1$$

$$\frac{3}{4}x^2+\frac{1}{4}y^2+\frac{\sqrt{3}}{2}xy+4\left(\frac{1}{4}x^2+\frac{3}{4}y^2-\frac{\sqrt{3}}{2}xy\right)=4$$

$$\frac{3}{4}x^2+\frac{1}{4}y^2+\frac{\sqrt{3}}{2}xy+x^2+3y^2-2\sqrt{3}xy-4=0$$

$$\frac{7}{4}x^2+\frac{13}{4}y^2-\frac{3}{2}\sqrt{3}xy-4=0$$

$$7x^2+13y^2-6\sqrt{3}xy-16=0$$

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$$5x^2 - 6xy + 5y^2 - 1 = 0$$

$a = \cos\alpha$

$b = \text{sen}\,\alpha$

$$\begin{cases} x = aX - bY \\ y = bX + aY \end{cases} \qquad \boxed{a^2 + b^2 = 1}$$

$$5(aX - bY)^2 - 6(aX - bY)(bX + aY) + 5(bX + aY)^2 - 1 = 0$$

$$5(a^2X^2 + b^2Y^2 - 2abXY) - 6(abX^2 + a^2XY - b^2XY - abY^2) + 5(b^2X^2 + a^2Y^2 +$$
$$2abXY) - 1 = 0$$

$$5a^2X^2 + 5b^2Y^2 \cancel{- 10abXY} - 6abX^2 - 6(a^2 - b^2)XY + 6abY^2 + 5b^2X^2 + 5a^2Y^2 +$$
$$\cancel{+ 10abXY} - 1 = 0$$

$$(5a^2 - 6ab + 5b^2)X^2 + (5b^2 + 6ab + 5a^2)Y^2 - 6(a^2 - b^2)XY - 1 = 0$$

$$\begin{cases} a^2 - b^2 = 0 \\ a^2 + b^2 = 1 \end{cases} \qquad \begin{cases} a^2 = b^2 \\ b = \pm \dfrac{\sqrt{2}}{2} \end{cases} \qquad a = \pm \frac{\sqrt{2}}{2}$$