EQUAZIONI OMOGENEE IN SEN X E COS X

1) $a\sin x + b\cos x = 0$ 1° grado.

2) $a\sin^2 x + b\sin x\cos x + c\cos^2 x = 0$ 2° grado.

3) $a\sin^3 x + b\sin^2 x\cos x + c\sin x\cos^2 x + d\cos^3 x = 0$ 3° grado.

} OMOGENEE

ESEMPIO

$\sin x + \cos x = 0$ pongo $\cos x \neq 0$ $x \neq \frac{\pi}{2} + k\pi$ $k \in \mathbb{N}$

divido per $\cos x$ e ottengo $\operatorname{tg} x + 1 = 0$ $\operatorname{tg} x = -1$ $\boxed{x = \frac{3}{4}\pi + k\pi \; k \in \mathbb{N}}$

se $x = \frac{\pi}{2} + k\pi \Rightarrow$ $1 + 0 = 0$ NO!

$-1 + 0 = 0$ NO!

$$S = \left\{ x \in \mathbb{R} \,/\, x = \frac{3}{4}\pi + k\pi,\ k \in \mathbb{N} \right\}.$$

ESEMPIO

$\sin^2 x + (1-\sqrt{3})\sin x \cos x - \sqrt{3}\cos^2 x = 0$ pongo $\cos x \neq 0 \Rightarrow x \neq \frac{\pi}{2} + k\pi \quad k \in \mathbb{N}$.

divido per $\cos^2 x$ e ho:

$$\operatorname{tg}^2 x + (1-\sqrt{3})\operatorname{tg} x - \sqrt{3} = 0$$

$$\operatorname{tg} x_{1,2} = \frac{-1+\sqrt{3} \pm \sqrt{1+3-2\sqrt{3}+4\sqrt{3}}}{2} = \frac{-1+\sqrt{3} \pm \sqrt{1+3+2\sqrt{3}}}{2} =$$

$$= \frac{-1+\sqrt{3} \pm (1+\sqrt{3})}{2} \rightarrow \frac{-1+\sqrt{3}-1-\sqrt{3}}{2} = -1$$

$$\rightarrow \frac{-1+\sqrt{3}+1+\sqrt{3}}{2} = \sqrt{3}$$

$\operatorname{tg} x = -1$ $\quad x = \frac{3}{4}\pi + k\pi \quad k \in \mathbb{N}$

$\operatorname{tg} x = \sqrt{3}$ $\quad x = \frac{\pi}{3} + k\pi \quad k \in \mathbb{N}$

se sostituisco nell'eq. di partenza $x = \frac{\pi}{2} + k\pi \quad k \in \mathbb{N}$ ottengo sempre $1 = 0$ NO!

ESEMPIO

$$\sin x \cos^2 x - \sqrt{3}\sin^2 x \cos x = 0$$

$$\sin x \cos x (\cos x - \sqrt{3}\sin x) = 0$$

$\sin x = 0 \quad x = k\pi \quad k \in \mathbb{N}$

$\cos x = 0 \quad x = \frac{\pi}{2} + k\pi \quad k \in \mathbb{N}$

$\cos x - \sqrt{3}\sin x = 0 \quad \operatorname{tg} x = \frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6} + k\pi, k \in \mathbb{N}$

EQUAZIONI RICONDUCIBILI AGLI OMOGENEE

$$a\sin^2 x + b\sin x\cos x + c\cos^2 x + d = 0$$

moltiplico il termine noto per $1 = \sin^2 x + \cos^2 x$

$$(a+d)\sin^2 x + b\sin x\cos x + (c+d)\cos^2 x = 0$$