FORMULE DI DUPLICAZIONE

$$\operatorname{sen} 2\alpha = \operatorname{sen}(\alpha+\alpha) = \operatorname{sen}\alpha \cos\alpha + \cos\alpha \operatorname{sen}\alpha = 2\operatorname{sen}\alpha\cos\alpha$$

$$\boxed{\operatorname{sen} 2\alpha = 2\operatorname{sen}\alpha\cos\alpha}$$

$$\cos 2\alpha = \cos(\alpha+\alpha) = \cos\alpha\cos\alpha - \operatorname{sen}\alpha\operatorname{sen}\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha$$

$$\boxed{\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha}$$

$$\operatorname{tg} 2\alpha = \operatorname{tg}(\alpha+\alpha) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\alpha}{1-\operatorname{tg}\alpha\operatorname{tg}\alpha} = \frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}$$

$$\boxed{\operatorname{tg} 2\alpha = \frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}}$$

$$\operatorname{ctg} 2\alpha = \operatorname{ctg}(\alpha+\alpha) = \frac{\operatorname{ctg}\alpha\operatorname{ctg}\alpha - 1}{\operatorname{ctg}\alpha + \operatorname{ctg}\alpha} = \frac{\operatorname{ctg}^2\alpha - 1}{2\operatorname{ctg}\alpha}$$

$$\boxed{\operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2\alpha - 1}{2\operatorname{ctg}\alpha}}$$

FORMULE DI BISEZIONE

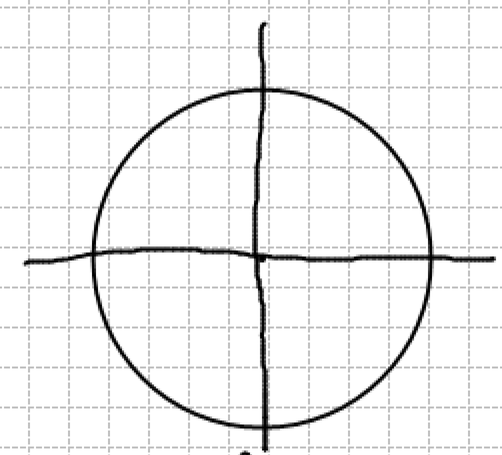
$\operatorname{sen}\frac{\alpha}{2} = ?$ $\quad \cos\frac{\alpha}{2} = ?$ $\quad \operatorname{Tg}\frac{\alpha}{2} = ?$ $\quad \operatorname{ctg}\frac{\alpha}{2} = ?$

$$\cos\alpha = \cos 2\left(\frac{\alpha}{2}\right) = \cos^2\frac{\alpha}{2} - \operatorname{sen}^2\frac{\alpha}{2} = \cos^2\frac{\alpha}{2} - 1 + \cos^2\frac{\alpha}{2} =$$

$$= 2\cos^2\frac{\alpha}{2} - 1$$

$$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$$

$$\boxed{\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}}$$



$$\cos\alpha = \cos 2\left(\frac{\alpha}{2}\right) = \cos^2\frac{\alpha}{2} - \operatorname{sen}^2\frac{\alpha}{2} = 1 - 2\operatorname{sen}^2\frac{\alpha}{2}$$

$$\operatorname{sen}^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$$

$$\boxed{\operatorname{sen}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}}$$

$$\boxed{\operatorname{Tg}\frac{\alpha}{2} = \frac{\operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}} \longleftrightarrow \boxed{\operatorname{Tg}\frac{\alpha}{2} = \frac{1-\cos\alpha}{\operatorname{sen}\alpha}} \ (\bullet)$$

$$\boxed{\operatorname{ctg}\frac{\alpha}{2} = \frac{\cos\frac{\alpha}{2}}{\operatorname{sen}\frac{\alpha}{2}} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}} \longleftrightarrow \boxed{\operatorname{ctg}\frac{\alpha}{2} = \frac{1+\cos\alpha}{\operatorname{sen}\alpha}}$$

$(\bullet)$ Dimostrazione

$$\operatorname{Tg}\frac{\alpha}{2} = \frac{\operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \frac{2\operatorname{sen}^2\frac{\alpha}{2}}{2\operatorname{sen}\frac{\alpha}{2}\cos\frac{\alpha}{2}} = \frac{2\operatorname{sen}^2\frac{\alpha}{2}}{\operatorname{sen}\alpha} = \frac{1-\cos\alpha}{\operatorname{sen}\alpha}$$

moltiplico per $2\operatorname{sen}\frac{\alpha}{2}$

$$\operatorname{ctg}\frac{\alpha}{2} = \frac{\cos\frac{\alpha}{2}}{\operatorname{sen}\frac{\alpha}{2}} = \frac{2\cos^2\frac{\alpha}{2}}{2\operatorname{sen}\frac{\alpha}{2}\cos\frac{\alpha}{2}} = \frac{1+\cos\alpha}{\operatorname{sen}\alpha}$$

moltiplico per $2\cos\frac{\alpha}{2}$

FORMULE PARAMETRICHE

$$\begin{cases} \operatorname{tg}\frac{\alpha}{2} = \dfrac{1-\cos\alpha}{\operatorname{sen}\alpha} \\ \operatorname{tg}\frac{\alpha}{2} = \dfrac{\operatorname{sen}\alpha}{1+\cos\alpha} \end{cases}$$

pongo $t = \operatorname{tg}\frac{\alpha}{2}$

$$\begin{cases} t \operatorname{sen}\alpha = 1-\cos\alpha \\ t(1+\cos\alpha) = \operatorname{sen}\alpha \end{cases}$$

$$\begin{cases} \cos\alpha = 1 - t\operatorname{sen}\alpha \\ t + t(1-t\operatorname{sen}\alpha) - \operatorname{sen}\alpha = 0 \end{cases}$$

$$\begin{cases} \cos\alpha = 1 - t\operatorname{sen}\alpha \\ t + t - t^2\operatorname{sen}\alpha - \operatorname{sen}\alpha = 0 \end{cases}$$

$$\begin{cases} \operatorname{sen}\alpha = \dfrac{2t}{1+t^2} \\ \cos\alpha = 1 - \dfrac{2t^2}{1+t^2} \end{cases}$$

$$\boxed{\begin{cases} \operatorname{sen}\alpha = \dfrac{2t}{1+t^2} \\ \cos\alpha = \dfrac{1-t^2}{1+t^2} \end{cases}}$$

$t = \operatorname{tg}\frac{\alpha}{2}$

$$\frac{\alpha}{2} \neq \frac{\pi}{2} + k\pi \quad k \in \mathbb{Z}$$

$$\alpha \neq \pi + 2k\pi \quad k \in \mathbb{Z}$$

FORMULE DI WERNER E PROSTAFERESI

$$\operatorname{sen}(\alpha+\beta)+\operatorname{sen}(\alpha-\beta)=2\operatorname{sen}\alpha\cos\beta$$

$$\operatorname{sen}(\alpha+\beta)-\operatorname{sen}(\alpha-\beta)=2\cos\alpha\operatorname{sen}\beta$$

$$\cos(\alpha+\beta)+\cos(\alpha-\beta)=2\cos\alpha\cos\beta$$

$$\cos(\alpha+\beta)-\cos(\alpha-\beta)=-2\operatorname{sen}\alpha\operatorname{sen}\beta$$

$$\cos\alpha\operatorname{sen}\beta=\frac{1}{2}\left[\operatorname{sen}(\alpha+\beta)-\operatorname{sen}(\alpha-\beta)\right]$$

$$\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$$

$$\operatorname{sen}\alpha\operatorname{sen}\beta=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$$

poniamo: $\begin{cases}\alpha+\beta=p\\ \alpha-\beta=q\end{cases}$ $\quad\begin{cases}2\alpha=p+q\\ 2\beta=p-q\end{cases}$ $\begin{cases}\alpha=\dfrac{p+q}{2}\\ \beta=\dfrac{p-q}{2}\end{cases}$

PROSTAFERESI:

$$\operatorname{sen}p+\operatorname{sen}q=2\operatorname{sen}\frac{p+q}{2}\cos\frac{p-q}{2}$$

$$\operatorname{sen}p-\operatorname{sen}q=2\cos\frac{p+q}{2}\operatorname{sen}\frac{p-q}{2}$$

$$\cos p+\cos q=2\cos\frac{p+q}{2}\cos\frac{p-q}{2}$$

$$\cos p-\cos q=-2\operatorname{sen}\frac{p+q}{2}\operatorname{sen}\frac{p-q}{2}$$

WERNER:

$$\cos\alpha\operatorname{sen}\beta=\frac{1}{2}\left[\operatorname{sen}(\alpha+\beta)-\operatorname{sen}(\alpha-\beta)\right]$$

$$\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right].$$

$$\operatorname{sen}\alpha\operatorname{sen}\beta=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right].$$