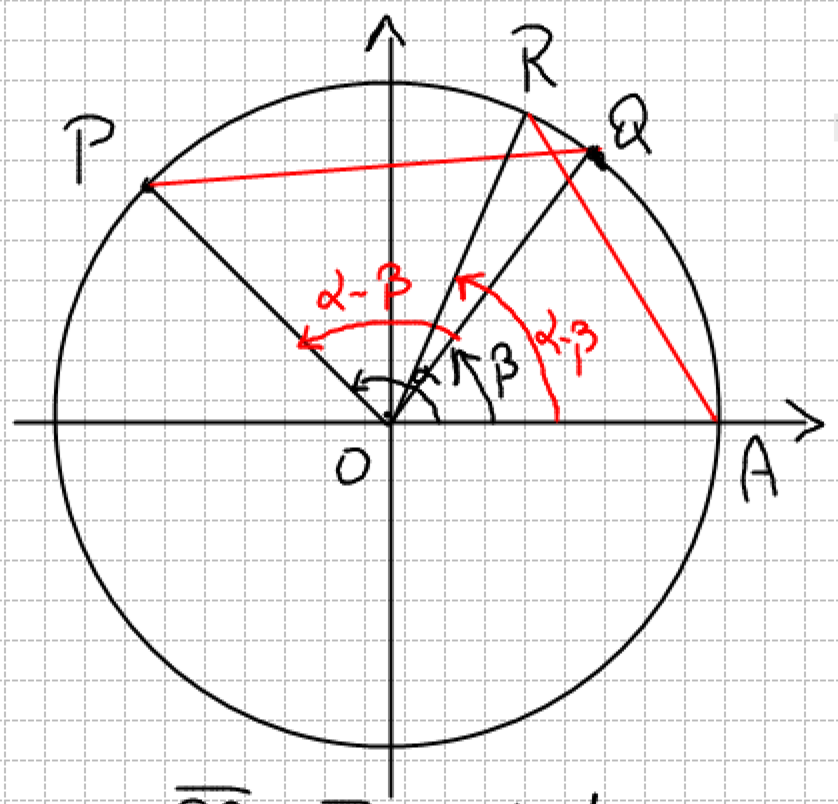
FORMULE DI ADDIZIONE E SOTTRAZIONE



$P(\cos\alpha;\ \operatorname{sen}\alpha)$

$Q(\cos\beta;\ \operatorname{sen}\beta)$

$R(\cos(\alpha-\beta);\ \operatorname{sen}(\alpha-\beta))$

$A(1;\ 0)$

$\overset{\triangle}{OAR} \cong \overset{\triangle}{QOP}$

Per costruzione $\overline{QP} = \overline{AR}$ quindi:

$$\sqrt{(\cos\alpha-\cos\beta)^2+(\operatorname{sen}\alpha-\operatorname{sen}\beta)^2} = \sqrt{(\cos(\alpha-\beta)-1)^2+\operatorname{sen}^2(\alpha-\beta)}$$

$$\underbrace{\cos^2\alpha}+\underbrace{\cos^2\beta}-2\cos\alpha\cos\beta+\underbrace{\operatorname{sen}^2\alpha}+\underbrace{\operatorname{sen}^2\beta}-2\operatorname{sen}\alpha\operatorname{sen}\beta = \underbrace{\cos^2(\alpha-\beta)}+1-2\cos(\alpha-\beta)+\underbrace{\operatorname{sen}^2(\alpha-\beta)}$$

$$1+1-2(\cos\alpha\cos\beta+\operatorname{sen}\alpha\operatorname{sen}\beta) = 1+1-2\cos(\alpha-\beta)$$

$$\cancel{2}-2(\cos\alpha\cos\beta+\operatorname{sen}\alpha\operatorname{sen}\beta) = \cancel{2}-2\cos(\alpha-\beta)$$

$$\boxed{\cos(\alpha-\beta) = \cos\alpha\cos\beta+\operatorname{sen}\alpha\operatorname{sen}\beta} \quad ✱ \quad (1)$$

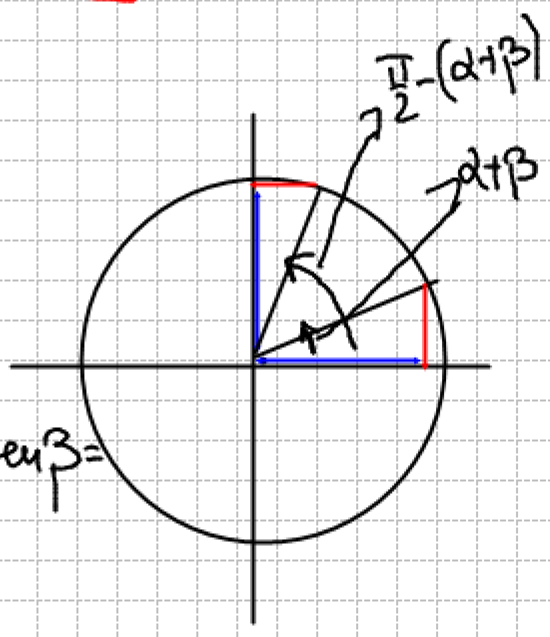
$$\cos(\alpha+\beta) = \cos(\alpha-(-\beta)) = \cos\alpha\cos(-\beta)+\operatorname{sen}\alpha\operatorname{sen}(-\beta) =$$

$$= \cos\alpha\cos\beta+\operatorname{sen}\alpha(-\operatorname{sen}\beta) = \cos\alpha\cos\beta-\operatorname{sen}\alpha\operatorname{sen}\beta.$$

$$\boxed{\cos(\alpha+\beta) = \cos\alpha\cos\beta-\operatorname{sen}\alpha\operatorname{sen}\beta} \quad ✱ \quad (2)$$

$$\operatorname{sen}(\alpha+\beta) = \cos\left[\frac{\pi}{2}-(\alpha+\beta)\right] =$$

$$= \cos\left[\boxed{\left(\frac{\pi}{2}-\alpha\right)}-\boxed{\beta}\right] = \cos\left(\frac{\pi}{2}-\alpha\right)\cos\beta+\operatorname{sen}\left(\frac{\pi}{2}-\alpha\right)\operatorname{sen}\beta =$$



$$= \operatorname{sen}\alpha\cos\beta+\cos\alpha\operatorname{sen}\beta$$

$$\boxed{\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha\cos\beta+\cos\alpha\operatorname{sen}\beta} \quad ✱ \quad (3)$$

$$\operatorname{sen}(\alpha-\beta) = \operatorname{sen}(\alpha+(-\beta)) = \operatorname{sen}\alpha\cos(-\beta)+\cos\alpha\operatorname{sen}(-\beta) =$$

$$= \operatorname{sen}\alpha\cos\beta-\cos\alpha\operatorname{sen}\beta$$

$$\boxed{\operatorname{sen}(\alpha-\beta) = \operatorname{sen}\alpha\cos\beta-\cos\alpha\operatorname{sen}\beta} \quad ✱ \quad (4)$$

$$Tg(\alpha+\beta)=\frac{sen(\alpha+\beta)}{cos(\alpha+\beta)}=\frac{sen\alpha\cos\beta+\cos\alpha\, sen\beta}{\cos\alpha\cos\beta-sen\alpha\, sen\beta}=(\cdot)$$

dividiamo numeratore e denominatore per $\cos\alpha\cos\beta\neq 0$, allora supponiamo $\cos\alpha\neq 0\Rightarrow \boxed{\alpha\neq\frac{\pi}{2}+k\pi \quad k\in\mathbb{Z}}$ e $\cos\beta\neq 0\Rightarrow$ $\boxed{\beta\neq\frac{\pi}{2}+h\pi \text{ con } h\in\mathbb{Z}}$

$$(\cdot)=\frac{\frac{sen\alpha\,\cancel{\cos\beta}}{\cos\alpha\,\cancel{\cos\beta}}+\frac{\cancel{\cos\alpha}\,sen\beta}{\cancel{\cos\alpha}\cos\beta}}{\frac{\cancel{\cos\alpha}\,\cancel{\cos\beta}}{\cancel{\cos\alpha}\,\cancel{\cos\beta}}-\frac{sen\alpha\, sen\beta}{\cos\alpha\,\cos\beta}}=\frac{Tg\alpha+Tg\beta}{1-Tg\alpha\, Tg\beta}$$

$$\boxed{Tg(\alpha+\beta)=\frac{Tg\alpha+Tg\beta}{1-Tg\alpha\, Tg\beta}}\quad * \quad (5)$$

$$Tg(\alpha-\beta)=Tg(\alpha+(-\beta))=\frac{Tg\alpha+Tg(-\beta)}{1-Tg\alpha\, Tg(-\beta)}=\frac{Tg\alpha-Tg\beta}{1+Tg\alpha\, Tg\beta}$$

$$\boxed{Tg(\alpha-\beta)=\frac{Tg\alpha-Tg\beta}{1+Tg\alpha\, Tg\beta}}\quad * \quad (6)$$

$$\operatorname{ctg}(\alpha+\beta)=\frac{\cos(\alpha+\beta)}{\operatorname{sen}(\alpha+\beta)}=\frac{\cos\alpha\cos\beta-\operatorname{sen}\alpha\operatorname{sen}\beta}{\operatorname{sen}\alpha\cos\beta+\cos\alpha\operatorname{sen}\beta}=(\cdot\cdot)$$

dividiamo numeratore e denominatore per $\operatorname{sen}\alpha\operatorname{sen}\beta\neq 0$, allora $\operatorname{sen}\alpha\neq 0 \Rightarrow \boxed{\alpha\neq k\pi,\ k\in\mathbb{Z}}$ e $\operatorname{sen}\beta\neq 0\Rightarrow \boxed{\beta\neq h\pi \text{ con } h\in\mathbb{Z}}$

$$(\cdot\cdot)=\frac{\dfrac{\cos\alpha\cos\beta}{\operatorname{sen}\alpha\operatorname{sen}\beta}-\dfrac{\cancel{\operatorname{sen}\alpha\operatorname{sen}\beta}}{\cancel{\operatorname{sen}\alpha\operatorname{sen}\beta}}}{\dfrac{\cancel{\operatorname{sen}\alpha}\cos\beta}{\cancel{\operatorname{sen}\alpha}\operatorname{sen}\beta}+\dfrac{\cos\alpha\cancel{\operatorname{sen}\beta}}{\operatorname{sen}\alpha\cancel{\operatorname{sen}\beta}}}=\frac{\operatorname{ctg}\alpha\operatorname{ctg}\beta-1}{\operatorname{ctg}\beta+\operatorname{ctg}\alpha}$$

$$\boxed{\operatorname{ctg}(\alpha+\beta)=\frac{\operatorname{ctg}\alpha\operatorname{ctg}\beta-1}{\operatorname{ctg}\beta+\operatorname{ctg}\alpha}}\quad * \quad (7)$$

$$\operatorname{ctg}(\alpha-\beta)=\operatorname{ctg}(\alpha+(-\beta))=\frac{\operatorname{ctg}\alpha\operatorname{ctg}(-\beta)-1}{\operatorname{ctg}(-\beta)+\operatorname{ctg}\alpha}=$$

$$=\frac{\operatorname{ctg}\alpha(-\operatorname{ctg}\beta)-1}{-\operatorname{ctg}\beta+\operatorname{ctg}\alpha}=\frac{1+\operatorname{ctg}\alpha\operatorname{ctg}\beta}{\operatorname{ctg}\beta-\operatorname{ctg}\alpha}$$

$\alpha\neq k\pi \quad k\in\mathbb{Z}$

$\beta\neq h\pi \quad h\in\mathbb{Z}$

$$\boxed{\operatorname{ctg}(\alpha-\beta)=\frac{1+\operatorname{ctg}\alpha\operatorname{ctg}\beta}{\operatorname{ctg}\beta-\operatorname{ctg}\alpha}}\quad * \quad (8)$$

$$\sec(\alpha+\beta) = \frac{1}{\cos(\alpha+\beta)} = \frac{1}{\cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta} =$$

$$= \frac{1}{\cos\alpha\cos\beta\left(1 - \operatorname{tg}\alpha\operatorname{tg}\beta\right)} = \frac{\sec\alpha\sec\beta}{1 - \sec\alpha\sec\beta\operatorname{sen}\alpha\operatorname{sen}\beta}$$

$$\operatorname{sen}\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{1}{\sec^2\alpha}} = \frac{\sqrt{\sec^2\alpha - 1}}{\sec\alpha}$$

$$\operatorname{cosec}(\alpha+\beta) = \frac{1}{\operatorname{sen}(\alpha+\beta)} = \frac{1}{\operatorname{sen}\alpha\cos\beta + \cos\alpha\operatorname{sen}\beta} =$$

$$= \frac{1}{\operatorname{sen}\alpha\operatorname{sen}\beta\left(\frac{\cos\beta}{\operatorname{sen}\beta} + \frac{\cos\alpha}{\operatorname{sen}\alpha}\right)} = \frac{\operatorname{cosec}\alpha\operatorname{cosec}\beta}{\operatorname{ctg}\beta + \operatorname{ctg}\alpha}$$

$$\operatorname{ctg}\beta = \operatorname{cosec}\beta\sqrt{1-\operatorname{sen}^2\beta} = \operatorname{cosec}\beta\sqrt{1-\frac{1}{\operatorname{cosec}^2\beta}}$$